Chapter - 13

OSCILLATION

WE LEARN ABOUT

13.1.1 Introduction

(Harmonic and non harmonic oscillation

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(Displacement, Velocity, Acceleration of SHM)

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Brief introduction

Hertz was born in Hamburg, Germany He studied science and engineering in the German cities. In 1880, Hertz obtained his PhD from the University of Berlin. In 1883, Hertz took a post as a lecturer in theoretical physics at the University of Kiel. In more advanced experiments, Hertz measure the velocity of electromagnetic radiation and found it to be the same as the light's velocity. He also showed that the nature of radio wave's reflection and refraction was the same as those of light, and established beyond any doubt that light is a form of electromagnetic radiation obeying the maxwell equations. His experiments would soon trigger the invention of the wireless telegraph and radio by marconi and other and tv. In recognition os his work, the unit of frequency - one cycle per second - is named the "hertz", in honor of heinrich hertz.

13.1.1 INTRODUCTION

In our daily life we come across various kinds of motions. You have already learnt about some of them, e.g. rectilinear motion and motion of a projectile. Both these motions are non-repetitive. We have also learnt about uniform circular motion and orbital motion of planets in the solar system. In these cases, the motion is repeated after a certain interval of time, that is, it is periodic. In your childhood you must have enjoyed rocking in a cradle or swinging on a swing. Both these motions are repetitive in nature but different from the periodic motion of a planet. Here, the object moves to and fro about a mean position. The pendulum of a wall clock executes a similar motion. There are leaves and branches of a tree oscillating in breeze, boats bobbing at anchor and the surging pistons in the engines of cars. All these objects execute a periodic to and fro motion. Such a motion is termed as oscillatory motion. In this chapter we study this motion.

The study of oscillatory motion is basic to physics; its concepts are required for the understanding of many physical phenomena. In musical instruments like the sitar, the guitar or the violin, we come across vibrating strings that produce pleasing sounds. The membranes in drums and diaphragms in telephone and speaker systems vibrate to and fro about their mean positions. The vibrations of air molecules make the propagation of sound possible. Similarly, the atoms in a solid oscillate about their mean positions and convey the sensation of temperature. The oscillations of electrons in the antennas of radio, TV and satellite transmitters convey information.

The description of a periodic motion in general, and oscillatory motion in particular, requires some fundamental concepts like period, frequency, displacement, amplitude and phase. These concepts are developed in the next section.

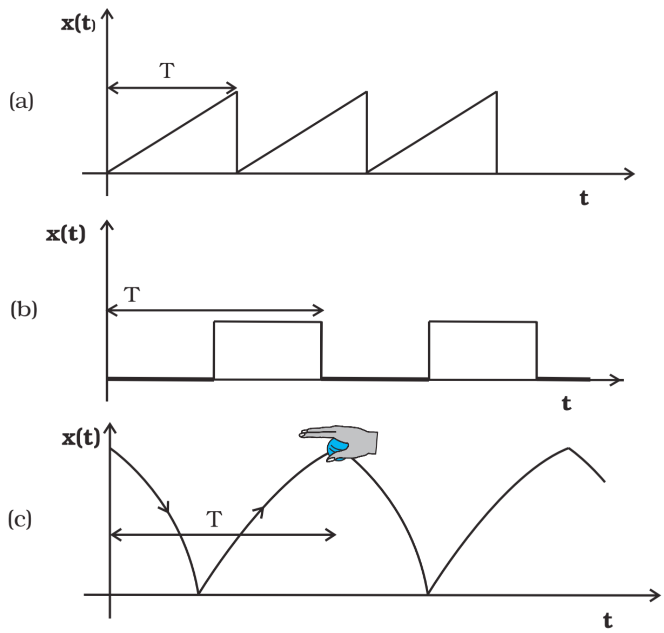
Periodic and oscillatory motions

Fig. shows some periodic motions. Suppose an insect climbs up a ramp and falls down it comes back to the initial point and repeats the process identically. If you draw a graph of its height above the ground versus time, it would look something like Fig.(a). If a child climbs up a step, comes down, and repeats the process, its height above the ground would look like that in Fig(b). When you play the game of bouncing a ball off the ground, between your palm and the ground, its height versus time graph would look like the one in Fig (c). Note that both the curved parts in Fig (c) are sections of a parabola given by the Newton's equation of motion.

$$h = ut + \frac{1}{2}gt^2 \text{ for downward motion, and}$$

$$h = ut - \frac{1}{2}gt^2 \text{ for upward motion,}$$

with different values of u in each case. These are examples of periodic motion. Thus, a motion that repeats itself at regular intervals of time is called periodic motion.



Examples of periodic motion. The period T is shown in each case.

Very often the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to oscillations or vibrations. For example, a ball placed in a bowl will be in equilibrium at the bottom. If displaced a little from the point, it will perform oscillations in the bowl. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory.

There is no significant difference between oscillations and vibrations. It seems that when the frequency is small, we call it oscillation (like the oscillation of a branch of a tree), while when the frequency is high, we call it vibration (like the vibration of a string of a musical instrument).

Simple harmonic motion is the simplest form of oscillatory motion. This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position, which is also the equilibrium position. Further, at any point in its oscillation, this force is directed towards the mean position.

In practice, oscillating bodies eventually come to rest at their equilibrium positions, because of the damping due to friction and other dissipative causes. However, they can be forced to remain oscillating by means of some external periodic agency. We discuss the phenomena of damped and forced oscillations later in the chapter.

Any material medium can be pictured as a collection of a large number of coupled oscillators. The collective oscillations of the constituents of a medium manifest themselves as waves. Examples of waves include water waves, seismic waves, electromagnetic waves.

Difference between periodic motion and oscillatory motion

All oscillatory motions are periodic motions because each oscillatory motion is completed in a definite interval of time. But all periodic motions may not be oscillatory. For example, the revolution of earth around the sun is a periodic motion but not an oscillatory motion, because the basic concept of to and fro or back and forth motion about some mean position for oscillatory motion is not present in this motion.

13.1.2 Harmonic oscillation

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (i.e., sine function or cosine function).

In such oscillations, when a body is displaced a little from its equilibrium position (i.e., mean position) and then left to itself, it begins to oscillate to and fro about its mean position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is proportional to the displacement of the body from the mean position at that instant. In the absence of frictional forces, the harmonic oscillation possesses constant amplitude. A harmonic oscillation of constant amplitude and of single frequency is called simple harmonic oscillation.

Mathematically, a simple harmonic oscillation can be expressed as

$$y = a \sin \omega t = a \sin 2 \pi t/T \quad ...(1)$$

or

$$y = a \cos \omega t = a \cos 2 \pi t/T \quad ...(2)$$

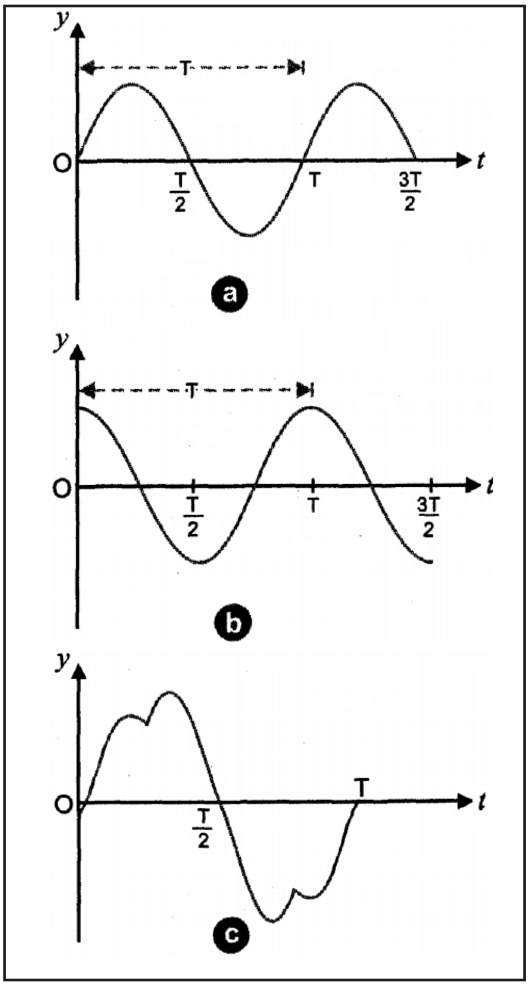
Here, y = displacement of body from mean position at any instant t.

a = maximum displacement or amplitude of displacement of the body.

ω = Angular frequency $(= 2 \pi v) = (2 \pi/T)$

v, T = frequency and time period of harmonic oscillation.

On plotting a graph between y and t as given by (1), we get a sine curve as shown in Fig (a).(a). Similarly, on plotting a graph between y and t given by (2), we get a cosine curve as shown in Fig.(b)



13.1.3 Non harmonic oscillation

Non-harmonic oscillation is that oscillation which cannot be expressed in terms of single harmonic function.

A non-harmonic oscillation is a combination of two or more than two harmonic oscillations.

Mathematically, non harmonic oscillation may be expressed as

$$y = a \sin \omega t + b \sin 2\omega t$$

$$y = a \sin \frac{2\pi}{T} t + b \sin \frac{4\pi t}{T} \quad ...(3)$$

Graphically, non-harmonic oscillation can be represented, by a curve of the type shown in Fig.(c).

13.1.4 Some important definitions related to periodic motion

(a) Time period: It is the least interval of time after which the periodic motion of a body repeats itself

It is denoted by T. If two sinusoidal functions with two different periods are of the type as shown in

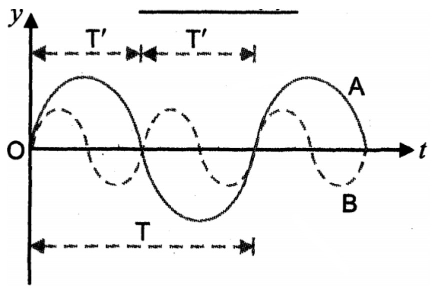


Fig. then the period of motion represented by curve A is T and that of B is T' (=T/2). SI unit of T is second.

(b) Frequency. It is defined as the number of periodic motions executed by body per second.

It is denoted by v. Clearly, v = 1/T. SI unit of v is hertz (denoted by Hz), where 1 Hz = 1 oscillation per second = 1 cps = 1 s^{-1} Note that the frequency v is not necessarily an integer.

(c) Angular frequency of a body executing periodic motion is equal to the product of frequency of the body with factor 2 π, i,e.,

Angular frequency, ω = v x 2 π v = 2π/T.SI unit of ω is rad/s.

(d) Displacement. In general, the name displacement is given to a change in physical quantity under consideration with time in a periodic motion. Thus, displacements represent changes in physical quantities with time such as position, angle, pressure, electric and magnetic fields etc.

Examples.

(i) In a loaded spring, When a body is oscillating under the action of a spring, the displacement variable is its deviation from the mean position of the oscillation, with time.

(ii) In a simple pendulum, the displacement variable is its angular deviation from the vertical during oscillations, with time.

(iii) During the propagation of sound wave in air, the displacement variable is the local change in pressure, with time.

(iv) During the propagation of electromagnetic waves, the displacement variables are electric and magnetic fields which vary periodically, with time.

Displacement variable is measured as a function of time, and it can have both positive and negative values. The displacement of an oscillating particle can be represented by x and y depending upon, whether the physical quantity changes along X-axis or Y-axis.

Displacement can be represented by a mathematical function of time. In a simple periodic motion, the displacement can be given by

$y = f(t) = a \sin \omega t$ or $x = f(t) = a \cos \omega t$

where a is called amplitude of oscillation or maximum displacement.

Note:

1. The maximum value of displacement of a particle while oscillating is called amplitude of oscillation.
2. The displacement of an oscillating particle can be linear or angular. The linear displacement is denoted by x or y and angular displacement by θ or ϕ
3. A body executing oscillatory motion is called an oscillator.

13.1.5 Periodic functions

Periodic functions are those functions which are used to represent periodic motion.

A function f(t) is said to be periodic, if

$$f(t) = f(t + T) = f(t + 2T) \quad ...(4)$$

Since, sine and cosine functions are the examples of periodic functions, therefore, a particle performing a periodic motion must return to its initial position after-one period of the motion. If T is the period of this periodic motion, then for periodic motion

$$y = a \sin \omega t = a \sin \omega(t + T) \quad ...(5)$$

or $$x = a \cos \omega t = a \cos \omega(t + T) \quad ...(6)$$

We know that the value of sine or cosine function repeats after a period of 2π radian.

$\therefore \quad \omega(t + T) = \omega t + 2\pi$ or $\omega T = 2\pi \quad ...(7)$

or $\quad \omega = 2\pi/T = 2\pi\nu \quad [\because (1/T) = \nu] \quad ...(8)$

where ω is called angular frequency.

The linear combination of sine and cosine functions is also a periodic function as discussed below.

Let a linear combination of sine and cosine functions be given by

$$x = f(t) = a \sin \omega t + b \cos \omega t \quad ...(9)$$

Taking, $a = R \cos \phi$ and $b = R \sin \phi \quad ...(10)$

Then, $x = R \cos \phi \sin \omega t + R \sin \phi \cos \omega t$

$$= R \sin (\omega t + \phi) \quad ...(11)$$

It represents a periodic function of time period T and amplitude R, where

$$R = \sqrt{a^2 + b^2} \quad ...(12)$$

and $\quad \tan \phi = b/a \quad ...(13)$

[Squaring and adding (10), we have, $R^2 (\cos^2 \phi + \sin^2 \phi) = a^2 + b^2$ or $R = \sqrt{a^2 + b^2}$; Dividing them, we have, $\tan \phi = b/a$]

Phase

Phase of a vibrating particle at any instant is a physical quantity which completely expresses the position and direction of motion of the particle at that instant with respect to its mean position.

It is measured either in terms of fraction of time period or fraction of 2π angle, which has elapsed since the vibrating particle has crossed its mean position in the positive direction.

In Oscillatory motion, the phase of a vibrating particle is the argument of sine or cosine function involved to represent the generalised equation of motion of the vibrating particle.

If the displacement of a particle at the instant of time t is represented by the equation

$$y = a \sin (\omega t + \theta_0) = a \sin(\frac{2\pi}{T} t + \phi_0),$$

then the quantity($= \frac{2\pi}{T} t + \phi_0$), is called phase of oscillation at time t. It is denoted by ϕ.

$$\phi = \frac{2\pi}{T} t + \phi_0 \quad ...(14)$$

Initial phase or epoch, it is the phase of a vibrating particle corresponding to time t = 0. When t = 0, from (14), $\phi = \phi_0$. Its unit is radian.

Note that the phase of a vibrating particle changes continuously with time but the epoch remains constant at all times.

Phase difference between two vibrating particles tells the lack of harmony in the vibrating states of the two particles at a given instant is measured as the difference in phase angles of the two vibrating particles at any instant.

Consider two particles A and B oscillating along a straight line with O as mean position with equal period but having different amplitudes.

(i) When the two vibrating particles cross their mean positions at the same time, moving in the same direction, the displacement (y) – time (t) graph for the motion of these two particles will be as shown in Fig. The motion of the two particles can be given by the equations

$$y_1 = a \sin \omega t$$
$$y_2 = b \sin \omega t$$

The phase difference between them is zero.

(ii) When the two vibrating particles cross their mean position at the same time, moving in the opposite directions and the particle A is ahead of particle B by half vibration, then the displacement-time graph of the motion of two particles, will be as shown in Fig. The motion of two particles can be represented by the equations :

$$y_1 = a \sin (\omega t + \pi), \ y_2 = a \sin \omega t$$

The phase difference between them is n rad or 180°.

(iii) When the particle A is passing from the extreme position with a further tendency to move towards left hand side and the particle B is passing from the mean position with a further tendency to move towards the right hand side, the particle A is ahead of particle B by 1/4 vibration. The displacement-time graph of the motion of the two particles will be as shown in Fig. The motion of the two particles will be represented by the equations

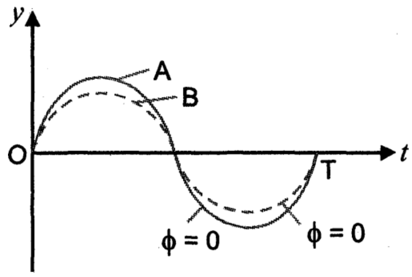
$$y_1 = a \sin (\omega t + \pi/2), \ y_2 = b \sin \omega t$$

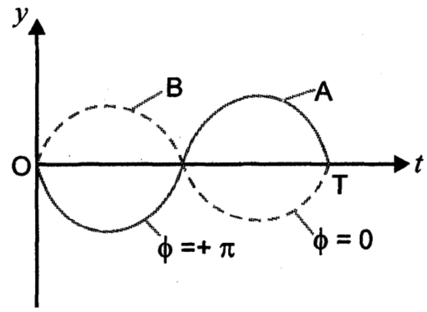
The phase difference between them is π/2 rad or 90°

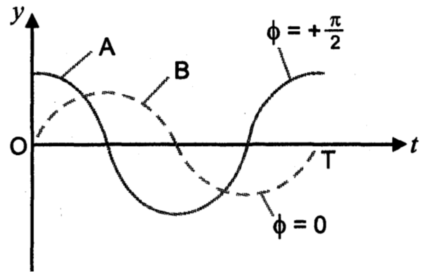
(iv) When the displacement-time graph of the motion of two particles is of the type as shown in Fig. and particle A is ahead of particle B by 1/8 vibration. The motions of two particles can be represented by

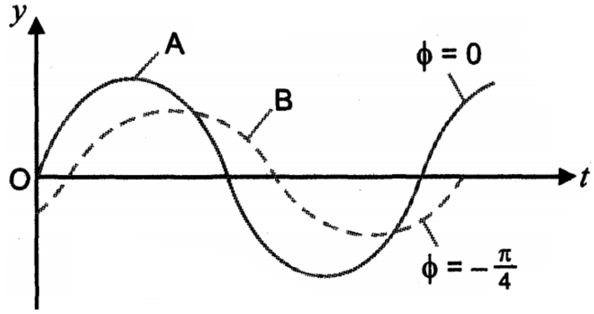
$$y_1 = a \sin \omega t, \ y_2 = b \sin (\omega t - \pi/4)$$

Now, the phase difference between them is π/4 rad or 45°.





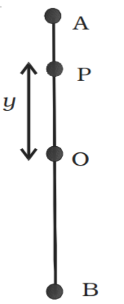




13.2.1 Simple harmonic motion

A particle is said to execute simple harmonic motion if its acceleration is directly proportional to the displacement from a fixed point and is always directed towards that point.

Consider a particle P executing SHM along a straight line between A and B about the mean position O (Fig).The acceleration of the particle is always directed towards a fixed point on the line and its magnitude is proportional to the displacement of the particle from this point.



Simple harmonic motion of a particle

(i.e) $a \propto y$

By definition $a = -\omega^2 y$

where ω is a constant known as angular frequency of the simple harmonic motion. The negative sign indicates that the acceleration is opposite to the direction of displacement. If m is the mass of the particle, restoring force that tends to bring back the particle to the mean position is given by

$$F = -m\omega^2 y$$

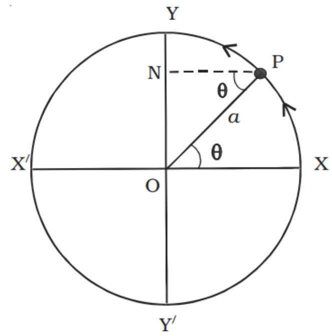
or $$F = -k y$$

The constant $k = m\omega^2$, is called force constant or spring constant. Its unit is N m^{-1}. The restoring force is directed towards the mean position.

Thus, simple harmonic motion is defined as oscillatory motion about a fixed point in which the restoring force is always proportional to the displacement and directed always towards that fixed point.

13.2.2 The projection of uniform circular motion on a diameter is SHM

Consider a particle moving along the circumference of a circle of radius a and centre O, with uniform speed v, in anticlockwise direction as shown in Fig. Let XX' and YY' be the two perpendicular diameters.

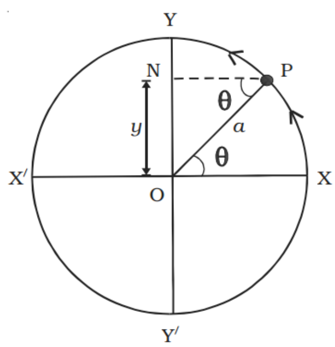


Suppose the particle is at P after a time t. If ω is the angular velocity, then the angular displacement θ in time t is given by $\theta = \omega t$. From P draw PN perpendicular to YY'. As the particle moves from X to Y, foot of the perpendicular N moves from O to Y. As it moves further from Y to X', then from X' to Y' and back again to X, the point N moves from Y to O, from O to Y' and back again to O. When the particle completes one revolution along the circumference, the point N completes one vibration about the mean position O. The motion of the point N along the diameter YY' is simple harmonic.

Hence, the projection of a uniform circular motion on a diameter of a circle is simple harmonic motion.

13.2.3 Displacement in SHM

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. When the particle is at P, the displacement of the particle along Y axis is y (Fig).



Then, in Δ OPN, $\sin\theta = ON/OP$

$$ON = y = OP \sin\theta$$

$$y = OP \sin\omega t \qquad (\because \theta = \omega t)$$

since OP = a, the radius of the circle, the displacement of the vibrating particle is

$$y = a \sin \omega t \qquad ...(1)$$

The amplitude of the vibrating particle is defined as its maximum displacement from the mean position.

13.2.4 Velocity in SHM

The rate of change of displacement is the velocity of the vibrating particle.

Differentiating eqn. (1) with respect to time t

$$\frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t)$$

$$\therefore \quad v = a\,\omega \cos \omega t \qquad ...(2)$$

The velocity v of the particle moving along the circle can also be obtained by resolving it into two components as shown in Fig.

(i) v cos θ in a direction parallel to OY

(ii) v sin θ in a direction perpendicular to OY

The component v sin θ has no effect along YOY′ since it is perpendicular to OY.

$\therefore$ Velocity = v cos θ

= v cos ωt

We know that, linear velocity = radius × angular velocity

$\therefore$ v = $a\omega$

$\therefore$ Velocity = aω cos ωt

$\therefore$ Velocity = $a\omega\sqrt{1-\sin^2\omega t}$

$$\text{Velocity} = a\omega\sqrt{1-\left(\frac{y}{a}\right)^2} \qquad \left[\because \sin\theta = \frac{y}{a}\right]$$

$$\text{Velocity} = \omega\sqrt{a^2 - y^2} \qquad ...(3)$$

13.2.5 Special cases

(i) When the particle is at mean position, (i.e) y = 0. Velocity is aω and is maximum. v = ± aω is called velocity amplitude.

(ii) When the particle is in the extreme position, (i.e) y = ± a, the velocity is zero.

13.2.6 Acceleration in SHM

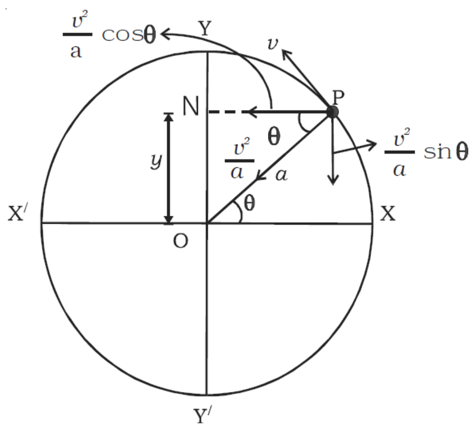
The rate of change of velocity is the acceleration of the vibrating particle.

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(a\omega \cos \omega t) = -\omega^2 a \sin \omega t.$$

$$\therefore \quad \text{acceleration} = \frac{d^2y}{dt^2} = -\omega^2 y$$

The acceleration of the particle can also be obtained by component method.

The centripetal acceleration of the particle P acting along PO is v^2/a. This acceleration is resolved into two components as shown in Fig.



Acceleration in SHM

(i) v^2/a cos θ along P N perpendicular to OY

(ii) v^2/a sin θ in a direction parallal to YO

The component v^2/a cos θ has no effect along YOY′ since it is perpendicular to OY.

Hence acceleration = – v^2/a sin θ

$= -a\,\omega^2 \sin \omega t \qquad (\because v = a\omega)$

$= -\omega^2 y \qquad (\because y = a \sin \omega t)$

$\therefore$ acceleration = $-\omega^2 y$

The negative sign indicates that the acceleration is always opposite to the direction of displacement and is directed towards the centre.

Special Cases

(i) When the particle is at the mean position (i.e) y = 0, the acceleration is zero.

(ii) When the particle is at the extreme position (i.e) y = ± a, acceleration is ∓ a ω^2 which is called as acceleration amplitude.

The differential equation of simple harmonic motion from eqn. (4) is $\frac{d^2y}{dt^2}\ \omega^2 + y = 0$ ...(5)

Using the above equations, the values of displacement, velocity and acceleration for the SHM are given in the Table .

It will be clear from the above, that at the mean position y = 0, velocity of the particle is maximum but acceleration is zero. At extreme position y = ± a, the velocity is zero but the acceleration is maximum $\mp a\omega^2$ acting in the opposite direction

Time	ωt	Displacement a sin ωt	Velocity aω cos ωt	Acceleration $-\omega^2 a \sin \omega t$
t = 0	0	0	$a\omega$	0
$t = \frac{T}{4}$	$\frac{\pi}{2}$	$+a$	0	$-a\omega^2$
$t = \frac{T}{2}$	π	0	$-a\omega$	0
$t = \frac{3T}{2}$	$\frac{3\pi}{2}$	$-a$	0	$+a\omega^2$
t = T	2π	0	$+a\omega$	0

13.2.7 Graphical representation of SHM

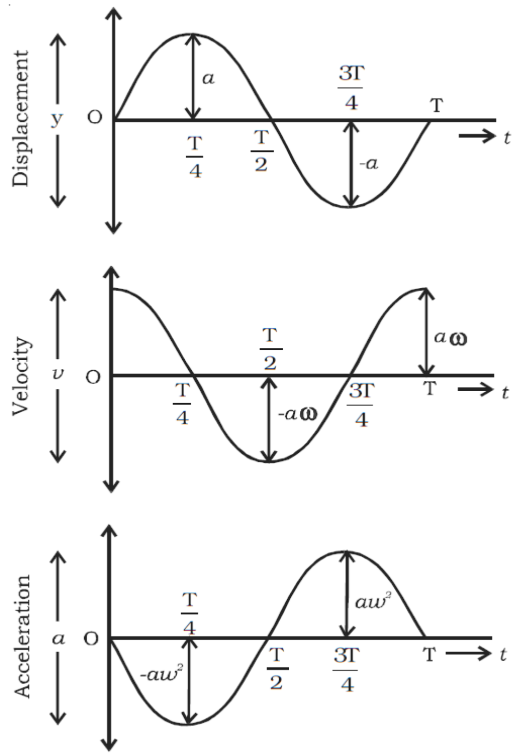
Graphical representation of displacement, velocity and acceleration of a particle vibrating simple harmonically with respect to time t is shown in Fig.

(i) Displacement graph is a sine curve. Maximum displacement of the particle is y = ± a.

(ii) The velocity of the vibrating particle is maximum at the mean position i.e v = ± a ω and it is zero at the extreme position.

(iii) The acceleration of the vibrating particle is zero at the mean position and maximum at the extreme position (i.e) $\mp a\omega^2$.

The velocity is ahead of displacement by a phase angle of π/2. The acceleration is ahead of the velocity by a phase angle π/2 or by a phase π ahead of displacement. (i.e) when the displacement has its greatest positive value, acceleration has its negative maximum value or vice versa.



Graphical representation

13.3.1 Energy in simple harmonic motion

The total energy (E) of an oscillating particle is equal to the sum of its kinetic energy and potential energy if conservative force acts on it.

The velocity of a particle executing SHM at a position where its displacement is y from its mean position is $v = \omega\sqrt{a^2 - y^2}$

Kinetic energy

Kinetic energy of the particle of mass m is

$$K = \frac{1}{2} m[\omega\sqrt{a^2 - y^2}\,]^2$$

$$K = \frac{1}{2}\ m\,\omega^2\,(a^2 - y^2) \qquad ..(1)$$

Potential energy

From definition of SHM F = –ky the work done by the force during the small displacement dy is dW = −F.dy = −(−ky) dy = ky dy

∴ Total work done for the displacement y is,

$$W = \int dW = \int_0^y ky\,dy$$

$$W = \int_0^y m\,\omega^2\,y\,dy \qquad [\because k = m\omega^2)$$

$$\therefore \quad W = \frac{1}{2} m\,\omega^2\,y^2$$

This work done is stored in the body as potential energy

$$U = \frac{1}{2} m \omega^2 y^2 \qquad ...(2)$$

Total energy E = K + U

$$= \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m \omega^2 y^2$$

$$= \frac{1}{2} m \omega^2 a^2$$

Thus we find that the total energy of a particle executing simple harmonic motion is $\frac{1}{2} m \omega^2 a^2$.

Special cases

(i) When the particle is at the mean position y = 0, from eqn (1) it is known that kinetic energy is maximum and from eqn. (2) it is known that potential energy is zero. Hence the total energy is wholly kinetic

$$E = K_{max} = \frac{1}{2} m \omega^2 a^2$$

(ii) When the particle is at the extreme position y = ± a, from eqn. (1) it is known that kinetic energy is zero and from eqn. (2) it is known that Potential energy is maximum. Hence the total energy is wholly potential.

$$E = U_{max} = \frac{1}{2} m \omega^2 a^2$$

(iii) When y = a/2,

$$K = \frac{1}{2} m \omega^2 \left[a^2 - \frac{a^2}{4}\right]$$

$$\therefore \quad K = \frac{3}{4}\left(\frac{1}{2} m\omega^2 a^2\right)$$

$$K = \frac{3}{4} E$$

$$U = \frac{1}{2} m \omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2} m\omega^2 a^2\right)$$

$$\therefore \quad U = \frac{1}{4} E$$

If the displacement is half of the amplitude, $K = \frac{3}{4}$ E and $U = \frac{1}{4}$ E. K and U are in the ratio 3 : 1,

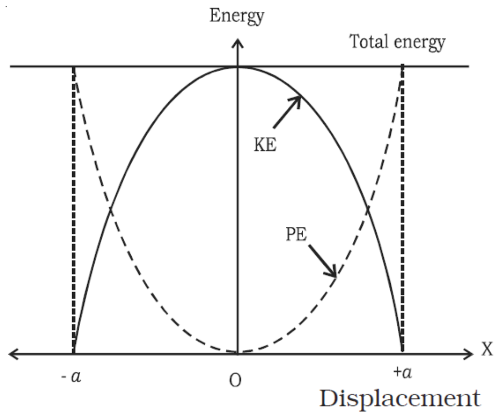
$$E = K + U = \frac{1}{2} m \omega^2 a^2$$

At any other position the energy is partly kinetic and partly potential.

This shows that the particle executing SHM obeys the law of conservation of energy.

Graphical representation of energy

The values of K and U in terms of E for different values of y are given in the Table. The variation of energy of an oscillating particle with the displacement can be represented in a graph as shown in the Fig.



Energy – displacement graph

y	0	a/2	a	– a/2	–a
Kinetic energy	E	$\frac{3}{4}$E	0	$\frac{3}{4}$E	0
Potential energy	0	$\frac{1}{4}$ E	E	$\frac{1}{4}$ E	E

Energy of SHM

13.3.2 Dynamics of harmonic oscillations

The oscillations of a physical system results from two basic properties namely elasticity and inertia. Let us consider a body displaced from a mean position. The restoring force brings the body to the mean position.

(i) At extreme position when the displacement is maximum, velocity is zero. The acceleration becomes maximum and directed towards the mean position.

(ii) Under the influence of restoring force, the body comes back to the mean position and overshoots because of negative velocity gained at the mean position.

(iii) When the displacement is negative maximum, the velocity becomes zero and the acceleration is maximum in the positive direction. Hence the body moves towards the mean position. Again when the displacement is zero in the mean position velocity becomes positive.

(iv) Due to inertia the body overshoots the mean position once again. This process repeats itself periodically. Hence the system oscillates.

The restoring force is directly proportional to the displacement and directed towards the mean position.

(i.e) $F \propto y$

$F = -ky$ ... (1)

where k is the force constant. It is the force required to give unit displacement. It is expressed in N m^{-1}.

From Newton's second law, F = ma ...(2)

$\therefore \quad -k\,y = ma$

or $\quad a = -\dfrac{k}{m}$ ...(3)

From definition of SHM acceleration $a = -\omega^2 y$

The acceleration is directly proportional to the negative of the displacement.

Comparing the above equations we get,

$$\omega = \sqrt{\frac{k}{m}} \quad ...(4)$$

Therefore the period of SHM is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi \times \sqrt{\frac{\text{inertial factor}}{\text{spring factor}}} \quad ...(5)$$

13.3.3 Angular harmonic oscillator

Simple harmonic motion can also be angular. In this case, the restoring torque required for producing SHM is directly proportional to the angular displacement and is directed towards the mean position.

Consider a wire suspended vertically from a rigid support. Let some weight be suspended from the lower end of the wire. When the wire is twisted through an angle θ from the mean position, a restoring torque acts on it tending to return it to the mean position. Here restoring torque is proportional to angular displacement θ.

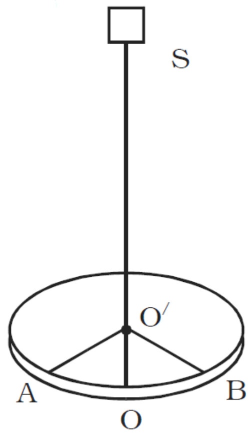
Hence $\quad \tau = -C\,\theta$ ...(1)

where C is called torque constant.

It is equal to the moment of the couple required to produce unit angular displacement. Its unit is N m rad^{-1}.

The negative sign shows that torque is acting in the opposite direction to the angular displacement. This is the case of angular simple harmonic motion.

Examples : Torsional pendulum, balance wheel of a watch.



Torsional Pendulum

But $\tau = I\,\alpha$ ...(2)

where τ is torque, I is the moment of inertia and α is angular acceleration

∴ Angular acceleration,

$$\alpha = \frac{\tau}{I} = -\frac{C\theta}{I} \quad ...(3)$$

This is similar to $\alpha = -\omega^2 y$

Replacing y by θ, and a by α we get

$$\alpha = -\omega^2\theta = -\frac{C}{I}\theta$$

$$\therefore \quad \omega = \sqrt{\frac{C}{I}}$$

$$\therefore \quad \text{Period of SHM } T = 2\pi\sqrt{\frac{C}{I}}$$

$$\therefore \quad \text{Frequency } n = \frac{1}{T} = \frac{1}{2\pi\sqrt{I/C}} = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

Case i

When a particle is performing SHM of time period T_1 the force acting on it is F_1 for a certain displacement. When the same particle is performing SHM of time period T_2 the force acting is F_2 for the same displacement. What will be the time period of the particle when a combined force of F_1 and F_2 produces the same displacement in SHM?

Sol:

The force actting on a paricle performing SHM is $F = -m\omega^2 x = -m\left(\dfrac{2\pi}{T}\right)^2 x$

When the force is F_1, $T = T_1$

$$\therefore \quad F_1 = -m\left(\frac{2\pi}{T_1}\right)^2 x$$

When the force is F_2, $T = T_2$

$$\therefore \quad F_2 = -m\left(\frac{2\pi}{T_2}\right)^2 x$$

When the force is $F_1 + F_2$, $T_1 = T_2$

$$\therefore \quad F_1 + F_2 = -m\left(\frac{2\pi}{T}\right)^2 x$$

$$-m\left(\frac{2\pi}{T_1}\right)^2 x - m\left(\frac{2\pi}{T_2}\right)^2 x = -m\left(\frac{2\pi}{T}\right)^2 x$$

$$\Rightarrow \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$

The time period of the particle,

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

Case ii:

In the above case if forces are acting simultaneously in opposite direction, then time period of particle is given by $F_1 - F_2 = F$.

$$-m\left(\frac{2\pi}{T_1}\right)^2 x + m\left(\frac{2\pi}{T_2}\right)^2 x = -m\left(\frac{2\pi}{T}\right)^2 x$$

$$\Rightarrow \frac{1}{T^2} = \frac{1}{T_1^2} - \frac{1}{T_2^2}$$

The time period of the particle,

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 - T_2^2}}$$

Case iii:

In the above case if forces are acting perpendicular to each other and acting simultaneously on the particle, then

$$F = \sqrt{F_1^2 + F_2^2} \qquad \frac{1}{T^2} = \sqrt{\left(\frac{1}{T_1^2}\right)^2 + \left(\frac{1}{T_2^2}\right)^2}$$

$$\frac{1}{T^4} = \frac{1}{T_1^4} + \frac{1}{T_2^4} \qquad T^4 = \frac{T_1^4 T_2^4}{T_1^4 + T_2^4}$$

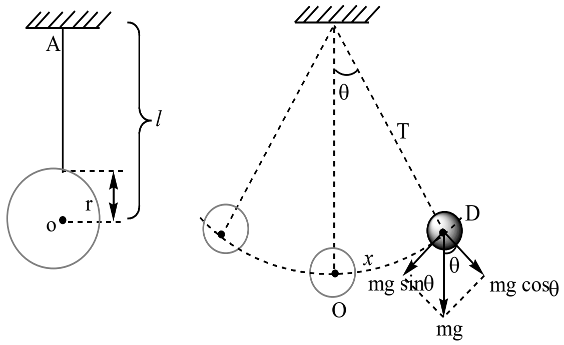
$$\therefore T = \left(\frac{T_1^4 T_2^4}{T_1^4 + T_2^4}\right)^4$$

13.3.4 Some systems executing SHM:

Simple pendulum:

A heavy point sized mass suspended by a massless inextensible, torsionless string is called an ideal "simple pendulum". But, in practice a small heavy metallic sphere (bob) suspended from oe end of a light thread is used as a simple pendulum. The distance between the point of suspension of the pendulum and the centre of gravity of the sphere is known as the "length of the pendulum(l)".

Expression for the time period of a simple pendulum:



OA = l = length of simple pendulum.

Consider a simple pendulum of length "l" carrying a bob of mass 'm'. Let the bob be given a small angular displacement and released. The bob oscillates to and fro about the equilibrium position 'O' in the vertical plane describing an arc of a circle.

At an instant, let the pendulum is making an angle 'θ' with the vertical and the bob is at 'D'. At this point there are only two forces acting on the bob; the tension T along the string and the weight of the bob mg act vertically downwards. The weight mg can be resolved in to two components. One of the component mg cos θ along the string and mg sin θ perpendicular to it. Since the motion of the bob is along a circle of radius l and centre at the support point, the bob has a radial acceleration ($\omega^2 l$) and also a tangental acceleration. The radial acceleration is provided by the net radial force T – mg cos θ, while the tangential acceleration is provided by mg sin θ. Torque τ about the support is entirely provided by the tangental component of force mg sin θ

$$\therefore \tau = -l(mg \sin\theta)$$

This is a restoring torque that tends to reduce angular displacement and hence the negative sign. By Newton's law of rotational motion, $\tau = I\alpha$

Where I is the moment of inertial of the system about the support and α is the angular acceleration

$$I\alpha = -mg\sin\theta\, l \text{ (or) } \alpha = -\frac{mgl}{I}\sin\theta$$

{For small values of θ, $\sin\theta \approx \theta$}

$$\therefore \quad \alpha = -\frac{mgl}{I}\theta$$

As the angular displacement θ increases sin θ largely differs form 'θ'. Therefore, for the oscillation to be along a straight line and for the condition of SHM that a ∞ –x to be valid θ must be very small.

From the above equation it is clear that angular acceleration is directly proportional to angular displacement and hence the motion is angular SHM for small values of θ

Comparing the above equation with $\alpha = -\omega^2\theta$.

We get

$$\omega = \sqrt{\frac{mgl}{I}} \quad T = 2\pi\sqrt{\frac{I}{mgl}}$$

Since the string of the simple pendulum is mass less, the moment of inertia I is simply ml^2

The time period of oscillation of the simple pendulum is given by

$$\therefore T = 2\pi\sqrt{\frac{l}{g}}$$

From the above equation it is clear that the time period of a simple pendulum does not depend on the mass, material, shape and size of the bob as long as the length of a simple pendulum at a given place remains same. For small angular amplitudes, time period does not depend on amplitude. For small angular amplitudes only, the oscillations of a simple pendulum are treated as SHM.

* The time period of oscillation of a pendulum depends on its length and acceleration due to gravity.

$$T \propto \sqrt{\frac{l}{g}} \text{ (or) } \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}\frac{g_2}{g_1}}$$

*<u>At a given place,</u> time period of a simple pendulum is directly proportional to the square root of its length.

$$T \propto \sqrt{l} \text{ (g is constant) } \frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$$

*<u>For a given length</u> time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}} \text{ (l is constant) } \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$$

*<u>For a given time period,</u> the length of the pendulum is directly proportional to acceleration due to gravity.

$$l \propto g \text{ (T is constant) } \frac{l_1}{l_2} = \sqrt{\frac{g_1}{g_2}}$$

The frequency of oscillations of he simple pendulum is given by

$$n = 1/T \qquad n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

From equation (5) the expression for acceleration due to gravity at a given place is given.

Seconds Pendulum:

A simple pendulum whose time period of oscillation is equal to two seconds, is called "seconds pendulum"

i.e., $T = 2s$ or $2\pi\sqrt{\frac{l}{g}} = 2$ or $l = \frac{g}{\pi^2}$

This is the length of a seconds pendulum and it changes from place to place.

If "g" is taken as $9.8\ ms^{-2}$, then the length of the second's pendulum is equal to

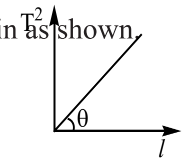
$$l = \frac{9.8}{(3.14)^2} = 0.994\ m (\because \pi^2 = 9.86) \text{ (or) } l \approx 1\ m$$

13.3.5 Graphs related to simple pendulum:

We have $T = 2\pi\sqrt{\frac{l}{g}}$ ot $T^2 = 4\pi^2\sqrt{\frac{l}{g}}$

and $\frac{l}{T^2}$ = constant a a given place.

If a graph is drawn taking length "l" on X – axis and time period "T^2" on Y – axis will be a straight line passing through origin as shown.

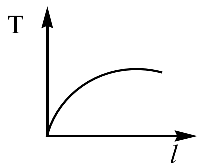


Slope of the graph "m"

$= \tan\theta = \frac{T^2}{l}$. from $T = 2\pi\sqrt{\frac{l}{g}}$

slope $= \frac{T^2}{l} = \frac{4\pi^2}{g}$. From this the value of 'g' at a given place can be found out.

If a graph is drawn taking length "l" on X – axis and time period T on Y – axis it will be a parabola as shown in figure.

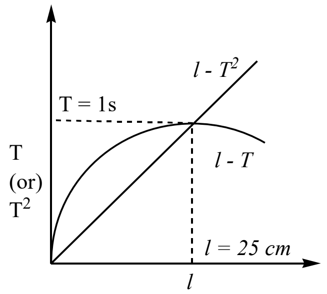


When $l - T$ & $l - T^2$ graph are plotted on the same graph paper they intersect at a pint as shown in the diagram. At the point of intersection,

$T = T^2$ $\therefore T = 1s.$

The length of the pendulum is given by

$$l = \frac{gT^2}{4\pi^2} = \frac{9.8(1)^2}{4(3.14)^2} \quad \text{(or)} \quad l = 25 \text{ cm.}$$



Therefore for T = 1s and $l = 25$ cm, $l - T$ & $l - T^2$ graphs intersect.

Special cases related to simple pendulum:

Let "T_1" and "T_2" are the time periods of oscillation of two simple pendulums of lengths "l_1" and "l_1" respectively.

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}} \quad \text{(or)} \quad l_1 = \frac{gT_1^2}{4\pi^2}$$

and $$T_2 = 2\pi\sqrt{\frac{l_2}{g}} \quad \text{(or)} \quad l_2 = \frac{gT_2^2}{4\pi^2}$$

If "T" is the time period of oscillation of a simple pendulum of length $(l_1 + l_2)$, then,

$$T = 2\pi\sqrt{\frac{l_1 + l_2}{g}} \quad \text{(or)} \quad l_1 + l_2 = \frac{gT^2}{4\pi^2}$$

$$\therefore \frac{gT_1^2}{4\pi^2} + \frac{gT_2^2}{4\pi^2} = \frac{gT^2}{4\pi^2} \qquad \therefore T_1^2 + T_2^2 = T^2$$

or $$T = \sqrt{T_1^2 + T_2^2}$$

In the similar lines one can show that if T is the time period oscillation of a simple pendulum of length, $[l_1 - l_2]$ then, $T = \sqrt{T_1^2 - T_2^2}$

Force(Spring) constant of a spring:

Consider a uniform, massless spring in its unstretched state.

When the spring is stretched, (or compressed) the restoring force developed in the spring tries to bring the spring back to its equilibrium position. The restring force developed us directly proportional to the elongation (or compression) produced in the spring.

If 'F' is the restoring force on producing an elongation (or) compression) 'x' in the spring, then,

$F \propto x$ (or) $F = -Kx$

The negative sign indicated, that the restoring force is in a direction opposite to 'x'. Here 'K' is a constant for a given spring and is called "Force constant" or "spring constant" or (stiffness constant" of the spring. It is given by

$$K = \frac{F}{x} = \frac{\text{restoring force}}{\text{elongation}}$$

"The restoring force developed per unit extension in the spring is called force constant of the spring".

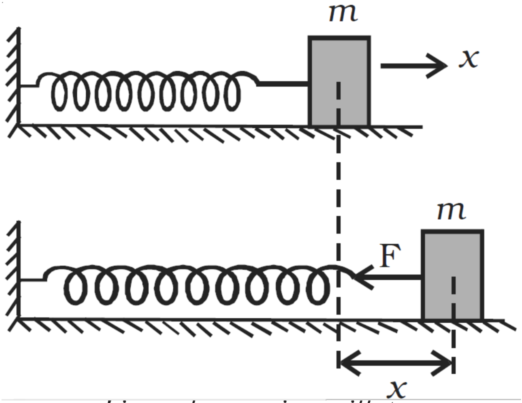
Its S.I unit is Nm^{-1} and C.G.S unit is dyne cm^{-1}

$1 \, Nm^{-1} = 10^3$ dyne cm^{-1}

Its dimensional formula is $= MT^{-2}$.

13.4.1 Horizontal oscillations of spring

Consider a mass (m) attached to an end of a spiral spring (which obeys Hooke's law) whose other end is fixed to a support as shown in Fig. The body is placed on a smooth horizontal surface. Let the body be displaced through a distance x towards right and released. It will oscillate about its mean position. The restoring force acts in the opposite direction and is proportional to the displacement.



Linear harmonic oscillator

$\therefore$ Restoring force F = −kx.

From Newton's second law, we know that F = ma

$\therefore$ ma = −kx

$a = \frac{-k}{m} x$

Comparing with the equation of SHM $a = -\omega^2 x$, we get

$\omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$

But $T = \frac{2\pi}{\omega}$

Time period $T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore$ Frequency $n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

13.4.2 Vertical oscillations of a spring

Fig.a. shows a light, elastic spiral spring suspended vertically from a rigid support in a relaxed position. When a mass 'm' is attached to the spring as in Fig.b, the spring is extended by a small length dl such that the upward force F exerted by the spring is equal to the weight mg.

The restoring force

$F = k\,dl$; $k\,dl = mg$ ...(1)

where k is spring constant. If we further extend the given spring by a small distance by applying a small force by our finger, the spring oscillates up and down about its mean position. Now suppose the body is at a distance y above the equilibrium position as in Fig.c. The extension of the spring is $(dl - y)$. The upward force exerted on the body is $k(dl - y)$ and the resultant force F on the body is

$F = k(dl - y) - mg = -ky$ ...(2)

The resultant force is proportional to the displacement of the body from its equilibrium position and the motion is simple harmonic.

If the total extension produced is $(dl + y)$ as in Fig.d the restoring force on the body is $k(dl + y)$ which acts upwards.

So, the increase in the upward force on the spring is $k(dl + y) - mg = ky$

Therefore if we produce an extension downward then the restoring force in the spring increases by ky in the upward direction. As the force acts in the opposite direction to that of displacement, the restoring force is − ky and the motion is SHM.

$F = -ky$

$ma = -ky$

$a = -\frac{k}{m} y$ ...(3)

$a = -\omega^2 y$ (expression for SHM)

Comparing the above equations, $\omega = \sqrt{\frac{k}{m}}$ ...(4)

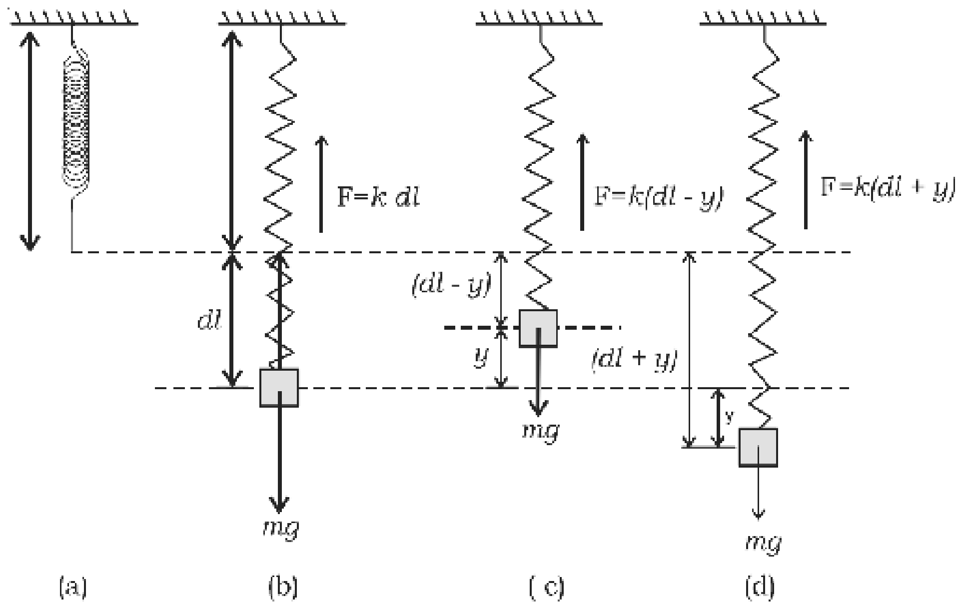
But $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ ...(5)

From equation (1) $mg = k\,dl$

$\frac{m}{k} = \frac{dl}{g}$

Therefore time period $T = 2\pi\sqrt{\frac{dl}{g}}$ ...(6)

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{g}{dl}}$



Vertical oscillations of loaded spring

Case 1 : When two springs are connected in parallel

Two springs of spring factors k_1 and k_2 are suspended from a rigid support as shown in Fig. A load m is attached to the combination.

Let the load be pulled downwards through a distance y from its equilibrium position. The increase in length is y for both the springs but their restoring forces are different.

If F_1 and F_2 are the restoring forces

$F_1 = -k_1 y,\ F_2 = -k_2 y$

$\therefore$ Total restoring force $= (F_1 + F_2) = -(k_1 + k_2)\,y$

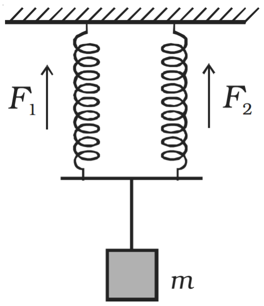
So, time period of the body is given by

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

If $k_1 = k_2 = k$

Then, $T = 2\pi\sqrt{\dfrac{m}{2k}}$

$\therefore$ frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}}$



13.4.3 Springs in Parallel:

Consider two massless springs of force constants 'K_1' and 'K_2' connected in parallel as shown in the figure. One end of the combination is connected to a rigid support and the other end to a block of mass "m".

We know that for parallel combination of springs, the effective spring constant is

$K = K_1 + K_1$

$\therefore$ Time period of oscillation in parallel combination is given by

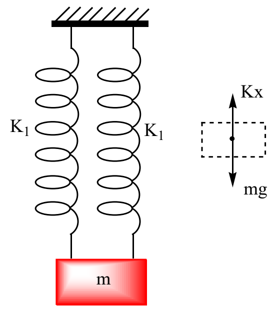
$$T = 2\pi\sqrt{\frac{m}{(K_1 + K_2)}}$$

Let T_1 and T_2 be the individual time period of oscillation for the same load and "T" is time period of oscillation of he parallel combination for the same load.

$\therefore T_1 = 2\pi\sqrt{\dfrac{m}{K_1}} \Rightarrow K_1 = \dfrac{C}{T_1^2}$

$T_2 = 2\pi\sqrt{\dfrac{m}{K_2}} \Rightarrow K_2 = \dfrac{C}{T_2^2}$

$T = 2\pi\sqrt{\dfrac{m}{K}} \Rightarrow K = \dfrac{C}{T^2}$



(Where C is a constant $= 4\pi^2.m$)

Since, $K = K_1 + K_2$

$$\frac{C}{T^2} = \frac{C}{T_1^2} + \frac{C}{T_2^2}$$

$$\therefore \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} = \frac{T_1^2 + T_2^2}{T_1^2 T_2^2}$$

$$\therefore T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

Case 2: When two springs are connected in series.

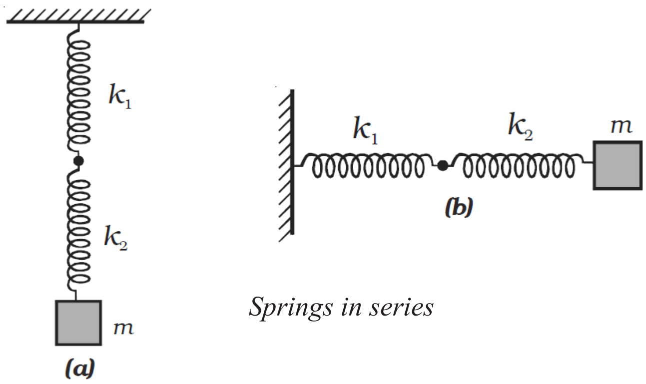
Two springs are connected in series in two different ways.

This arrangement is shown in Fig. a and b.

In this system when the combination of two springs is displaced to a distance y, it produces extension y_1 and y_2 in two springs of force constants k_1 and k_2.

$F = -k_1 y_2\ ;\ F = -k_2 y_2$

where F is the restoring force.



Springs in series

Total extension, $y = y_1 + y_2 = -F\left[\dfrac{1}{k_1} + \dfrac{1}{k_2}\right]$

We know that $F = -ky$

$\therefore \quad y = -\dfrac{F}{k}$

From the above equations,

$$-\frac{F}{k} = -F\left[\frac{1}{k_1} + \frac{1}{k_2}\right] \text{or } k = \frac{k_1 k_2}{k_1 + k_2}$$

$\therefore$ Time period $= T = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_1 k_2}{(k_1 + k_2)\,m}}$

If both the springs have the same spring constant,

$k_1 = k_2 = k.$

$\therefore \quad n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{2m}}$

Springs in series:

Consider two massless springs of force constant 'K_1' and 'K_2' connected in series as shown is the figure. One end of the combination is connected to a rigid support and the other end to a block of mass "m".

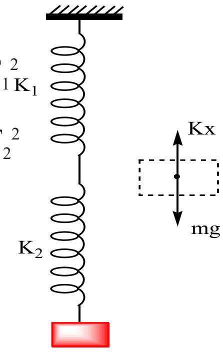
We know that for springs in series, the effective spring constant is

$$K = \frac{K_1K_2}{K_1+K_2} \quad (or) \quad \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

∴ Time period of oscillation in series combination is given by $T = 2\pi\sqrt{\frac{m}{K}}$

$$T = 2\pi\sqrt{\frac{m(K_1+K_2)}{K_1K_2}}$$

Let T_1 and T_2 be the individual time periods of oscillation of the two springs for the same load and 'T' is the time period of oscillation of the series combination for the same load.



Then, $T_1 = 2\pi\sqrt{\frac{m}{K_1}} \Rightarrow \frac{1}{K_1} = C.T_1^2$

$T_2 = 2\pi\sqrt{\frac{m}{K_2}} \Rightarrow \frac{1}{K_2} = C.T_2^2$

$T = 2\pi\sqrt{\frac{m}{K}} \Rightarrow \frac{1}{K} = C.T^2$

(Where C is a constant = $4\pi^2 m$)

But, $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

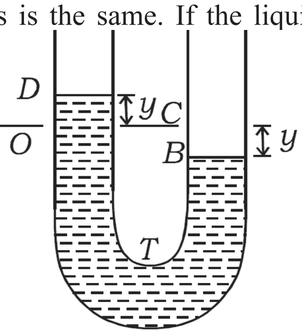
∴ $\cancel{C}T^2 = \cancel{C}T_1^2 + \cancel{C}T_2^2$

(or) $T^2 = T_1^2 + T_2^2$

∴ $T = \sqrt{T_1^2 + T_2^2}$

13.4.4 Oscillation of liquid column in a U – tube

Consider a non viscous liquid column of length l of uniform cross-sectional area A (Fig. Initially the level of liquid in the limbs is the same. If the liquid on one side of the tube is depressed by blowing gently the levels of the liquid oscillates for a short time about their initial positions O and C, before coming to rest.



If the liquid in one of the limbs is depressed by y , there will be a difference of 2 y in the liquid levels in the two limbs. At some instant, suppose the level of the liquid on the left side of the tube is at D, at a height y above its original position O, the level B of the liquid on the other side is then at a depth y below its original position C. So the excess pressure P on the liquid due to the restoring force is excess height × density × g

(i.e) pressure = $2\,y\,\rho\,g$

∴ Force on the liquid = pressure × area of the cross – section of the tube

$$= -2\,y\,\rho\,g \times A \qquad ..... (1)$$

The negative sign indicates that the force towards O is opposite to the displacement measured from O at that instant.

The mass of the liquid column of length l is volume × density

(i.e) $m = l\,A\,\rho$

∴ $F = l\,A\,\rho\,a \qquad .... (2)$

From equations (1) and (2) $l\,A\,\rho\,a = -\,2\,y\,A\,\rho\,g$

∴ $a = -\frac{2g}{l}y \qquad ..... (3)$

We know that $a = -\,\omega^2 y$

(i.e) $a = -\frac{2g}{l}y = -\omega^2 y$ where $\omega = \sqrt{\frac{2g}{l}}$

Here, the acceleration is proportional to the displacement, so the motion is simple harmonic and the period T is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{2g}}$$

13.4.5 Damped simple harmonic motion

So far we have dealt oscillation of a body which are simple harmonic, in which total energy of oscillation is constant.

But when the oscillations are in a medium which offer some resistance force, the amplitude and energy gradually decreases with time and fin ally the oscillation will be stopped, oscillations are called "Damped Oscillations".

In dampled oscillations, the energy of system is dissipated continuously; but, for small damping, the oscillations remain approximately periodic, the dissipating forces are generally the frictional forces.

Consider the oscillations of a particle of mass 'm ' in a medium which offers resistance to the motion. Let 'x' be the displacement of the particle from its equilibrium at any instant 't' and 'v' be the velocity of the particle at that instant.

The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping w ill be much greater and the dissipation of energy much faster. The damping force is generally proportional to velocity of the body. Opposite to the direction of velocity. If the damping force is denoted by F_d,

$$F_d = -bv$$

Where 'b' is damping coefficient (positive constant) depends on characteristics of the medium (viscosity) and size and shape of the block.

The total restoring force on the particle at the position is

$$F_R = -(kx + bv) \Rightarrow ma = -(kx + bv)$$

$$\therefore \frac{d^2x}{dt^2} + \frac{b}{m}v + \frac{k}{m}v = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}v = 0$$

The solution of the above equation is $x = A\, e^{-bt/2m} Cos(\omega t+\phi)$ where A is the amplitude and ω' is the angular frequency of the damped oscillator and its value is

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

The function x is not strictly periodic because of the factor $e^{-bt/2m}$ which decreases continuously with time.

However, if the decrease is small in one time period T, the motion is approximately periodic.

The mechanical energy of the damped oscillator is

$$E = -\frac{1}{2} KA^2 e^{-bt/2m}$$

The above equation shows that the total energy of the system decreases exponentially with time

NOTE:

- When b =0, all equations of damped oscillator reduce to the equations of an undamped oscillator.
- a) Small damping means that the dimension less ratio $\frac{b}{\sqrt{km}} << 1$ or $b << \sqrt{km}$
- b) When $b^2 < 4$ mk the system is said to be at the state of under damping.
- c) When $b^2 > 4$ mk, the system is said to be at the states or over damping.
- d) When $b^2 = 4$ mk, the system is said to be at the state of critical damping.

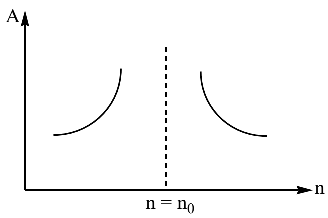
FORCED OSCILLATIONS AND RESONANCE:

Forced vibrations

If a body is made to vibrate under the influence of an external periodic force, such that it vibrates with the frequency of the periodic force impressed on it, such oscillations are known as forced vibrations. The natural frequency of the vibrating body need not be equal to the frequency of the periodic force. The amplitude of vibration is finite and constant. It depends on frequency of applied force, body and damping. Lesser the difference in frequencies and lesser the damping, greater will be the amplitude of vibration. The resultant displacement of the body is not in phase (lead or lag) with the applied force.

RESONANCE VIBRATIONS:

In the absence of damping "if a body is made to vibrate under the influence of an external periodic force and if the natural frequency of that body coincides with the frequency of the periodic force impressed on it, that body vibrates with increasing amplitude." This phenomenon is known as "resonance".

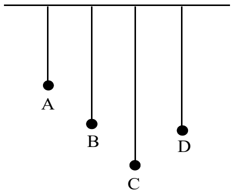


Resonance is a special case of forced vibrations with $\gamma = 0$ and $\omega_0 = \omega_e$. Theoretically at resonance amplitude of vibration is infinite. Resonant force vibrations are also known as sympathetic vibrations. The following examples explain about resonance.

e.g.(i) When we tune a particular station on our transistor or TV we make the frequency of our set equal to that of the desired station so that due to resonance it catches the desired station. The amplitude of vibration becomes maximum. As a result the signal is heard louder.

e.g.(ii) Resonance is not desired sometimes. When a band of soldiers are marching on a bridge, they are asked to go out of step. If the soldiers march in step, the amplitude of vibrations of the bridge may increase enormously. So, bridge may vibrate violently and collapse.

e.g.(iii) Consider four pendulums A, B, C and D suspended from a stretched rubber cord as shown. Here pendulums B and D have the same length. A is shorter than D, and C is longer than D. Let D be set into oscillations (by pulling it to a side and released).



The oscillations of D are free or natural oscillations. Then due to the transmission through the rubber cord, forced oscillations take place in all the three remaining i.e., A, B and C. So, these three pendulums start oscillating initially. The oscillations in A and C die down faster where as B and D will have sustained oscillations with larger amplitude. In this case B and D are in resonance since both have the same length and same natural frequency. Here their amplitude increase gradually. The oscillations of A and C are forced oscillations which die down faster.

Exercises – 1

Displacement of S.H.M, and Phase

1. The motion of a particle is given by
$x = A \sin \omega t + B \cos \omega t$.
The motion of the particle is
[1] Not simple harmonic
[2] Simple harmonic with amplitude A + B
[3] Simple harmonic with amplitude (A + B) / 2
[4] Simple harmonic with amplitude $\sqrt{A^2 + B^2}$

2. A particle starts S.H.M. from the mean position. Its amplitude is A and time period is T. At the time when its speed is half of the maximum speed, its displacement y is
[1] A/2 [2] $A/\sqrt{2}$
[3] $A\sqrt{3}/2$ [4] $2A/\sqrt{3}$

3. Two points are located at a distance of 10 m and 15 m from the source of oscillation. The period of oscillation is 0.05 sec and the velocity of the wave is 300 m/sec. What is the phase difference between the oscillations of two points
[1] π [2] π/6
[3] π/3 [4] 2π/6.

4. Two equations of two S.H.M. are $y = a\sin(\omega t - \alpha)$ and $y = b\cos(\omega t - \alpha)$. The phase difference between the two is
[1] 0° [2] α° [3]90° [4] 180°

5. The equation of a simple harmonic wave is given by $y = 6\sin 2\pi(2t - 0.1x)$,where x and y are in mm and t is in seconds. The phase difference between two particles 2 mm apart at any instant is
[1] 54° [2]72° [3] 18° [4] 36°

6. The equation of S.H.M. is $y = a\sin(2\pi nt + \alpha)$, then its phase at time t is
[1] $2\pi nt$ [2] α
[3] $2\pi nt + \alpha$ [4] $2\pi t$

7. A simple harmonic oscillator oscillates, with an amplitude A. At what point of its motion, is the power delivered to it by the restoring force maximum
[1] When it is at a displacement $\pm A/\sqrt{2}$ from the equilibrium point and moving towards the equilibrium point
[2] When it is at the maximum displacement
[3]When it passes through the equilibrium point, either way
[4] When it is at a displacement $\pm A/\sqrt{2}$ from the equilibrium point moving away from the equilibrium point

8. A simple harmonic oscillator has an amplitude α and time period T, The time required by it to travel from x = a to x =a/2 is
[1]T/6 [2]T/4 [3] T/3 [4] T/2

9. The displacement of a particle along the x axis is given by $x = a\sin^2\omega t$. The motion of the particle corresponds to
[1] Simple harmonic motion of frequency ω/2 π
[2] Simple harmonic motion of frequency ω/π
[3] Simple harmonic motion of frequency 3ω/2 π
[4] Non simple harmonic motion

10. A 1.00 x 10^{-20} kg particle is vibrating with simple harmonic motion with a period of 1.00 x 10^{-5} s and a maximum speed of 1.00 x 10^{-3}m/s . The maximum displacement of the particle is
[1] 1.59 mm [2] 1.00 m
[3] 10 m [4] None of these

11. The phase (at a time t) of a particle in simple harmonic motion tells
[1] Only the position of the particle at time t
[2]Only the direction of motion of the particle at time t
[3] Both the position and direction of motion of the particle at time t
[4] Neither the position of the particle nor its direction of motion at time t

12. A particle is moving with constant angular velocity along the circumference of a circle. Which of the following statements is true
[1] The particle so moving executes S.H.M.
[2]The projection of the particle on any one of the diameters executes S.H.M.

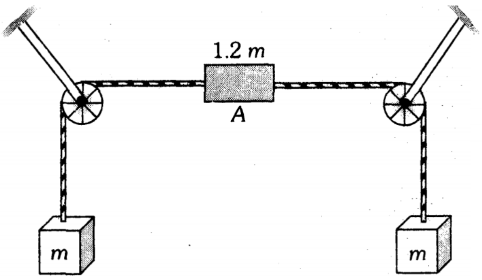
[3] The projection of the particle on any of the diameters executes S.H.M.
[4] None of the above

13. A particle is executing simple harmonic motion with a period of T seconds and amplitude a metre. The shortest time it takes to reach a point $a/\sqrt{2}$ m from its mean position in seconds is
[1] T [2] T/4 [3] T/8 [4] T/16

14. A simple harmonic motion is represented by F(t) = 10 sin (20t + 0.5). The amplitude of the S.H.M. is
[1] a = 30 [2] a = 20
[3] a = 10 [4] a = 5

15. Which of the following equations does not represent a simple harmonic motion
[1] y = a sin ωt [2] y = a cos ωt
[4] y = a sin ωt + b cos ωt
[4] y = a tan ωt

16. In the figure, the vertical sections of the string are long. A is released from rest from the position shown. Then



[1] The system will remain in equilibrium
[2] The central block will move down continuously
[3] The central block will undergo simple harmonic motion
[4] The central block will undergo periodic motion but not simple harmonic motion.

17. A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to go directly from its mean position to half the amplitude
[1] T/2 [2] T/ 4
[3] T /8 [4] T/12

18. A particle executing simple harmonic motion along y-axis has its motion described by the equation y = A sin(ωt) + B . The amplitude of the simple harmonic motion is
[1] A [2] B
[3] A + B [4] $\sqrt{A+B}$

19. The equation describing the motion of a simple harmonic oscillator along the x axis is given as : x = Acos(ωt + ϕ). If at time t = 0, the oscillator is at x = 0 and moving in the negative x direction, then the phase angle ϕ is
[1] π/2 [2] –π/2 [3] π [4] 0

20. Which one of the following is a simple harmonic motion
[1]Wave moving through a string fixed at both ends
[2] Earth spinning about its own axis
[3] Ball bouncing between two rigid vertical walls
[4] Particle moving in a circle with uniform speed

21. A particle is moving in a circle with uniform speed. Its motion is
[1] Periodic and simple harmonic
[2] Periodic but not simple harmonic
[3] Aperiodic [4] None of the above

22. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
[1] –π/3 [2] π/6
[3] –π/6 [4] π/3

23. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

[1] For an amplitude of g/ω^2
[2] For an amplitude of g^2/ω^2
[3] At the highest position of the platform
[4] At the mean position of the platform

24. The periodic time of a body executing simple harmonic motion is 3 s. After how much interval from time t = 0, its displacement will be half of its amplitude

[1] $\frac{1}{8}$ s [2] $\frac{1}{6}$ s
[3] $\frac{1}{4}$ s [4] $\frac{1}{3}$ s

25. Two simple harmonic motions are given by $x_1 = a \sin \omega t + a \cos \omega t$ and $x_2 = a \sin \omega t + \frac{a}{\sqrt{3}} \cos \omega t$ The ratio of the amplitudes of first and second motion and the phase difference between them are respectively

[1] $\sqrt{\frac{3}{2}}$ and $\frac{\pi}{12}$ [2] $\frac{\sqrt{3}}{2}$ and $\frac{\pi}{12}$
[3] $\frac{2}{\sqrt{3}}$ and $\frac{\pi}{12}$ [4] $\sqrt{\frac{3}{2}}$ and $\frac{\pi}{6}$

26. The differential equation of a particle executing SHM along y – axis is

[1] $\frac{d^2y}{dt^2} + \omega^2 y = 0$ [2] $\frac{d^2y}{dt^2} + \omega^2 y^2 = 0$
[3] $\frac{d^2y}{dt^2} - \omega^2 y = 0$ [4] $\frac{d^2y}{dt^2} + \omega y = 0$

27. The restoring force of SHM is maximum when particle

[1] Displacement is maximum
[2] Is half way between the mean and extreme position
[3] Crosses mean position
[4] Is at rest

28. A particle moves in x – y plane according to rule x = a sin ωt and y = a cos ωt. The particle follows

[1] An elliptical path [2] A circular path
[3] A parabolic path
[4] A straight line path inclined equally to x and y – axis

29. Out of the following functions representing motion of a particle which represents SHM

(a) $y = \sin \omega t - \cos \omega t$
(b) $y = \sin^3 \omega t$
(c) $y = 5 \cos\left(\frac{3\pi}{4} - 3\omega t\right)$
(d) $y = 1 + \omega t + \omega^2 t^2$

[1] Only (1) and (2)
[2] Only (1)
[3] Only (4) does not represent SHM
[4] Only (1) and (3)

30. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is

[1] π/2 [2] π/3
[3] π/4 [4] π/6

31. The motion which is not simple harmonic is

[1] Vertical oscillations of a spring
[2] Motion of simple pendulum
[3] Motion of a planet around the sun
[4] Oscillation of liquid column in a U-tube
[5] Vertical oscillation of a wooden plank floating in a liquid

Velocity of Simple Harmonic Motion

1. A simple pendulum performs simple harmonic motion about X = 0 with an amplitude A and its period T. The speed of the pendulum at X = A/2 will be

[1] $\frac{\pi A \sqrt{3}}{T}$ [2] $\frac{\pi A \sqrt{3}}{T}$
[3] $\frac{\pi A \sqrt{3}}{T}$ [4] $\frac{\pi A \sqrt{3}}{T}$

2. A body is executing simple harmonic motion with an angular frequency 2 rad/s. The velocity of the body at 20 mm displacement, when the amplitude of motion is 60 mm, is

[1] 40 mm /s [3] 60 mm/s
[3] 113 mm/s [4] 120 mm/s

3. The displacement equation of a particle is x = 3 sin2t + 4cos2t. The amplitude and maximum velocity will be respectively
[1] 5,10 [2] 3,2
[3] 4, 2 [4] 3 ,4

4. A simple harmonic oscillator has a period of 0.01 s and an amplitude of 0.2 m. The magnitude of the velocity in m sec^{-1} at the centre of oscillation is
[1] 20π [2] 100
[3] 40π [4] 100π

5. A particle executes S.H.M. with a period of 6 second and amplitude of 3 cm. Its maximum speed in cm/s is
[1] $\pi/2$ [2] π [3] 2π [4] 3π

6. The maximum velocity of a simple harmonic motion represented by $y = 3 \sin (100t + \pi/6)$ is given by
[1] 300 [2] $3\pi/6$
[3] 100 [4] $\pi/6$

7. A S.H.M. has amplitude 'a' and time period T. The maximum velocity will be
[1] 4a/T [2] 2a/T
[3] $2\pi\sqrt{\frac{a}{T}}$ [4] $\frac{2\pi a}{T}$

8. A body is executing S.H.M. When its displacement from the mean position is 4 cm and 5 cm, the corresponding velocity of the body is 10 cm/s and 8 cm/s. Then the time period of the body is
[1] 2πs [2] $\pi/2$s
[3] π s [4] $3\pi/2$s

9. The equation of a simple harmonic wave is given by
$y = 3 \sin \frac{\pi}{2}(50t - x)$
Where x and y are in meters and t is in seconds. The ratio of maximum particle velocity to the wave velocity is
[1] 2π [2] $3/2\ \pi$
[3] 3π [4] $2/3\ \pi$

10. If a simple pendulum oscillates with an amplitude of 50 mm and time period of 2 s, then its maximum velocity is
[1] 0.10 m/s [2] 0.15 m /s
[3] 0.8 m/s [4] 0.26 m /s

11. If the displacement of a particle executing SHM is given by $y = 0.30 \sin(220t + 0.64)$ in metre, then the frequency and maximum velocity of the particle is
[1] 35 Hz, 66 m/s [2] 45 Hz, 66 m/s
[3] 58 Hz, 113 m/s [4] 35 Hz, 132 m/s

12. A particle executes simple hamonic motion with a time period of 16s. At time t = 2s, the particle crosses the mean position while at t = 4s, its velocity is $4 ms^{-1}$. The amplitude of motion in metre is
[1] $\sqrt{2}\ \pi$ [2] $16\sqrt{2}\ \pi$
[3] $24\sqrt{2}\ \pi$ [4] $32\sqrt{2}\ /\pi$

13. The amplitude of a particle executing SHM is 4 cm. At the mean position the speed of the particle is 16 cm/s. The distance of the particle from the mean position at which the speed of the particle becomes $8\sqrt{3}$ cm / s, will be
[1] $2\sqrt{3}$ cm [2] $\sqrt{3}$ cm
[3] 1 cm [4] 2 cm.

14. The maximum velocity of a particle executing SHM is V. If the amplitude is doubled and the time period of oscillation decreased to 1/3 of its original value, the maximum velocity, becomes
[1] 18 V [2] 12 V
[3] 6 V [4] 3 V

15. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is
[1] $\sqrt{3}$ cm [2] $\sqrt{5}$ cm
[4] $2(\sqrt{3})$cm [4] $2(\sqrt{5})$ cm

16. Two particles P and Q start from origin and execute Simple Harmonic Motion along X-axis with same amplitude but with periods 3 seconds

and 6 seconds respectively. The ratio of the velocities of P and Q when they meet is

[1] 1:2 [2] 2 : 1

[3] 2 :3 [4] 3 :2

17. The instantaneous displacement of a simple pendulum oscillator is given by $x = A\cos(\omega t + \pi/4)$. Its speed will be maximum at time

[1] $\pi/4\omega$ [2] $\pi/2\omega$

[3] π/ω [4] $2\pi/\omega$

18. The angular velocities of three bodies in simple harmonic motion are ω_1, ω_2, ω_3, with their respective amplitudes as A_1, A_2, A_3. If all the three bodies have same mass and velocity, then

[1] $A_1\omega_1 = A_2\omega_2 = A_3\omega_3$

[1] $A_1\omega_1^2 = A_2\omega_2^2 = A_3\omega_3^2$

[1] $A_1^2\omega_1 = A_2^2\omega_2 = A_3^2\omega_3$

[1] $A_1^2\omega_1^2 = A_2^2\omega_2^2 = A^2$

19. The velocity of a particle performing simple harmonic motion, when it passes through its mean position is

[1] Infinity [2] Zero

[3] Minimum [4] Maximum

20. The velocity of a particle in simple harmonic motion at displacement y from mean position is

[1] $\omega\sqrt{a^2+y^2}$ [2] $\omega\sqrt{a^2-y^2}$

[3] ωy [4] $\omega^2\sqrt{a^2-y^2}$

21. A particle is executing the motion $x = A\cos(\omega t - 0)$. The maximum velocity of the particle is

[1] $A\omega\cos\theta$ [2] $A\omega$

[3] $A\omega\sin\theta$ [4] None of these

22. Velocity at mean position of a particle executing S.H.M. is υ, they velocity of the particle at a distance equal to half of the amplitude

[1] 4υ [2] 2υ

[3] $\sqrt{3}/2\ \upsilon$ [4] $\sqrt{3}/4\ \upsilon$

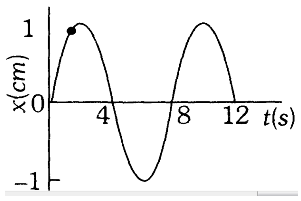
23. A body of mass M suspended from two springs separately executes simple harmonic motion. During oscillation the maximum velocity is equal in both cases. The ratio of amplitude a

[1] k_1/k_2 [2] k_2/k_1

[3] $\sqrt{k_2/k_1}$ [4] k_1^2/k_2^2

ACCELERATION OF SIMPLE HARMONIC MOTION

1. The x – t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is



[1] $\frac{\sqrt{3}}{32}\pi^2$ cm/s²

[2] $\frac{-\pi^2}{32}$ cm/s²

[3] $\frac{\pi^2}{32}$ cm/s

[4] $-\frac{\sqrt{3}}{32}\pi^2$ cm/s²

2. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following does not change with time

[1] $a^2T^2 + 4\pi^2 v^2$ [2] aT/x

[3] $aT + 2\pi v$ [4] aT/u

3. A particle oscillating under a force $F = -k\vec{x} - b\vec{v}$ is a (k and b are constants)

[1] Simple harmonic oscillator

[2] Non linear oscillator

[3] Damped oscillator

[4] Forced oscillator [5] Linear oscillator

4. Which one of the following equations of motion represents simple harmonic motion

[1] Acceleration $= -k_0x + k_1x^2$

[2] Acceleration $= -k(x + a)$

[3] Acceleration $= k(x + a)$

[4] Acceleration $= kx$

Where k, k_0, k_1 and a are all positive

5. The amplitude of a particle executing S.H.M. with frequency of 60 Hz is 0.01 m. The maximum value of the acceleration of the particle is

[1] $144 \pi^2$ m/s^2 [2] 144 m/s^2
[3] $144/\pi^2$ m/s^2 [4] $288 \pi^2$ m/s^2

6. Two simple harmonic motions of angular frequency 100 and 1000 rad s^{-1} have the same displacement amplitude. The ratio of their maximum accelerations is
[1] $1:10^3$ [2] $1:10^4$
[3] 1:10 [4] $1: 10^2$

7. A body executing simple harmonic motion has a maximum acceleration equal to 24 metres/s^2 and maximum velocity equal to 16 metres/s. The amplitude of the simple harmonic motion is
[1] 32/3 metres [2] 3/32 metres
[3] 1024/9 metres [4] 64/9 metres

8. For a particle executing simple harmonic motion, which of the following statements is not correct
[1] The total energy of the particle always remains the same
[2] The restoring force is always directed towards a fixed point
[3] The restoring force is maximum at the extreme positions
[4] The acceleration of the particle is maximum at the equilibrium position

9. A particle of mass 10 grams is executing simple harmonic motion with an amplitude of 0.5 m and periodic time of (π/5) seconds. The maximum value of the force acting on the particle is
[1] 25 N [2] 5 N
[3] 2.5 N [4] 0.5 N

10. A simple pendulum is suspended from the ceiling of a lift. When the lift is at rest its time period is T. With what' acceleration should the lift be accelerated upwards in order to reduce its period to T/2 (g is acceleration due to gravity)
[1] 4 g [2] g
[3] 2g [4] 3 g

11. A particle moving along the x-axis executes simple harmonic motion, then the force acting on it is given by
[1] – A Kx [2] A cos(Kx)
[3] A exp (– Kx) [4] A Kx
Where A and K are positive constants

12. A body is vibrating in simple harmonic motion with an amplitude of 0.06 m and frequency of 15 Hz. The velocity and acceleration of body is
[1] 5.65 m/s and 5.32 x 10^2m/s^2
[2] 6.82 m/s and 7.62 x 10^2m/s^2
[3] 8.91 m/s and 8.21 x 10^2m/s^2
[4] 9.82m/s and 9.03 x 10^2m/s^2

13. A particle executes harmonic motion with an angular velocity and maximum acceleration of 3.5 rad/s and 7.5 m/s^2 respectively. The amplitude of oscillation is
[1] 0.28 m [2] 0.36 m
[3] 0.53 m [4] 0.61 m

14. A 0.10 kg block oscillates back and forth along a horizontal surface. Its displacement from the origin is given by: x = (10cm)cos[(10rad/s)t + π/2 rad]. What is the maximum acceleration experienced by the block
[1] 10 m/s^2 [2] 10π m/s^2
[3] 10π /2 m/s^2 [4] $10\pi/3$ m/s^2

15. In S.H.M. maximum acceleration is at
[1] Amplitude [2]Equilibrium
[3] Acceleration is constant
[4] None of these

16. A particle executing simple harmonic motion with amplitude of 0.1 m. At a certain instant when its displacement is 0.02 m, its acceleration is 0.5 m/s^2. The maximum velocity of the particle is (in m/s)
[1] 0.01 [2] 0.05
[3] 0.5 [4] 0.25

17. Acceleration of a particle, executing SHM, at it's mean position is
[1] Infinity [2] Varies
[3] Maximum [4] Zero

18. Which one of the following statements is true for the speed v and the acceleration a of a particle executing simple harmonic motion
[1] When v is maximum, a is maximum
[2] Value of a is zero, whatever may be the value of v
[3]When vis zero, a is zero
[4] When vis maximum, a is zero

19. What is the maximum acceleration of the particle doing the SHM y = 2 sin $\left[\frac{\pi t}{2}+\phi\right]$ where 2 is in cm
[1] $\pi/2$ cm/s^2 [2] $\pi^2/2$ cm/s^2
[3] $\pi/4$ cm/s^2 [4] $\pi^2/4$ cm/s^2

20. A particle executes linear simple harmonic motion with an amplitude of 2 cm. When the particle is at 1 cm from the mean position the magnitude of its velocity is equal to that of its acceleration. Then its time period in seconds is
[1] $1/2\pi\sqrt{3}$ [2] $2\pi\sqrt{3}$
[3] $2\pi/\sqrt{3}$ [4] $\sqrt{3}/2\pi$

21. In simple harmonic motion, the ratio of acceleration of the particle to its displacement at any time is a measure of
[1] Spring constant
[2] Angular frequency
[3] (Angular frequency)^2
[4] Restoring force

22. The maximum velocity and the maximum acceleration of a body moving in a simple harmonic oscillator are 2 m/s and 4 m/s^2. Then angular velocity will be
[1] 3 rad/s [2] 0.5 rad/s
[3] 1 rad/s [4] 2 rad/s

23. A particle is performing simple harmonic motion with amplitude A and angular velocity ω. The ratio of maximum velocity to maximum acceleration is
[1] ω [2] 1/ ω [3] ω^2 [4] A ω.

24. The SHM of a particle is given by X(t) = 5 cos $\left(2\pi t+\frac{\pi}{4}\right)$ (in MKS units). Calculate the displacement and the magnitude of acceleration of the particle at t = 1.5 seconds.
[1] – 3.0m, 100 m/s^2 [2] + 2.54m, 200 m/s^2
[3] – 3.54m, 140m/s^2 [4] + 3.55m, 120 m/s^2

Energy of Simple Harmonic Motion

1. The total energy of a particle executing S.H.M. is proportional to
[1] Displacement from equilibrium position
[2] Frequency of oscillation
[3] Velocity in equilibrium position
[4] Square of amplitude of motion

2. A particle is oscillating in SHM, What fraction of total energy is kinetic when the particle is at A/2 from the mean position (A is the amplitude of oscillation)
[1] 3/4 [2] 2/4
[3] 4/7 [4] 5/7

3. A particle is vibrating in a simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position, is its energy half potential and half kinetic
[1] 1 cm [2] $\sqrt{2}$ cm
[3] 3 cm [4] 2 $\sqrt{2}$ cm.

4. If a body is executing simple harmonic motion, then
[1] At extreme positions, the total energy is zero
[2] At equilibrium position, the total energy is in the form of potential energy
[3] At equilibrium position, the total energy is in the form of kinetic energy
[4] At extreme positions, the total energy is infinite

5. A body performs SHM with an amplitude A. At a distance $A/\sqrt{2}$ from the mean position, the correct relation between KE and PE is
[1] KE = PE/2 [2] KE = $\sqrt{2}$ PE
[3] KE = PE [4] KE = PE/$\sqrt{2}$

6. The kinetic energy and potential energy of a particle executing simple harmonic motion will be equal, when displacement (amplitude = a) is
[1] a/2 [2] a$\sqrt{2}$
[3] a/$\sqrt{2}$ [4] a$\sqrt{2}$/3

7. The total energy of the body executing S.H.M. is E. Then the kinetic energy when the displacement is half of the amplitude, is
[1] E/2 [2] E/4
[3] 3E/4 [4] $\sqrt{3}$/4 E

8. The potential energy of a particle executing S.H.M. is 2.5 J, when its displacement is half of amplitude. The total energy of the particle be
[1] 18 J [2] 10 J
[3] 12 J [4] 2.5 J

9. The angular velocity and the amplitude of a simple pendulum is ω and a respectively. At a displacement X from the mean position if its kinetic energy is T and potential energy is V, then the ratio of T to V is
[1] $X^2\omega^2/(a^2 - X^2\omega^2)$ [2] $X^2/(a^2 - X^2)$
[3] $(a^2 - X^2\omega^2)\,X^2\omega^2$ [4] $(a^2 - X^2)/X^2$

10. When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is
[1] a/4 [2] a/3
[3] a/2 [4] 2a/3

11. The time period of the variation of potential energy of a particle executing SHM with period T is
[1] T/4 [2] T [3] 2T
[4] T/2 [5] T/3.

12. When the displacement is half the amplitude, the ratio of potential energy to the total energy is
[1] 1/2 [2] 1/4
[3] 1 [4] 1/8

13. The P.E. of a particle executing SHM at a distance x from its equilibrium position is
[1] $\frac{1}{2}m\omega^2x^2$ [2] $\frac{1}{2}m\omega^2a^2$
[3] $\frac{1}{2}m\omega^2(a^2 - x^2)$ [4] Zero

14. A vertical mass-spring system executes simple harmonic oscillations with a period of 2 s. A quantity of this system which exhibits simple harmonic variation with a period of 1 s is
[1] Velocity [2] Potential energy
[3] Phase difference between acceleration and displacement
[4] Difference between kinetic energy and potential energy

15. For any S.H.M., amplitude is 6 cm. If instantaneous potential energy is half the total energy then distance of particle from its mean position is
[1] 3 cm [2] 4.2 cm
[3] 5.8 cm [4] 6 cm

16. A body of mass 1 kg is executing simple harmonic motion. Its displacement y(cm) at t seconds is given by y = 6sin(100t+π4) . Its maximum kinetic energy is
[1] 6 J [2] 18 J
[3] 24 J [4] 36 J.

17. A particle is executing simple harmonic motion with frequency f. The frequency at which its kinetic energy change into potential energy is
[1] $f/2$ [2] f [3] $2f$ [4] $4f$

18. There is a body having mass m and performing S.H.M. with amplitude a. There is a restoring force F = –Kx , where x is the displacement. The total energy of body depends upon
[1] K, x [2] K a
[3] K, a, x [4] K, a, v

19. The total energy of a particle executing S.H.M. is 80 J. What is the potential energy when the particle is at a distance of 3/4 of amplitude from the mean position

[1] 60 J [2] 10 J
[3] 40 J [4] 45 J

20. In a simple harmonic oscillator, at the mean position
[1] Kinetic energy is minimum, potential energy is maximum
[2] Both kinetic and potential energies are maximum
[3] Kinetic energy is maximum, potential energy is minimum
[4] Both kinetic and potential energies are minimum

21. Displacement between maximum potential energy position and maximum kinetic energy position for a particle executing S.H.M. is
[1] – a [2] + a
[3] ± a [4] ± a/4

22. When a mass M is attached to the spring of force constant k, then the spring stretches by l. If the mass oscillates with amplitude l, what will be maximum potential energy stored in the spring
[1] $kl/2$ [2] $2kl$
[3] $\frac{1}{2}Mgl$ [4] Mgl.

23. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (where E is the total energy)
[1] 1/8 E [2] 1/4 E
[3] 1/2 E [4] 2/3 E

24. A body executes simple harmonic motion. The potential energy (P.E.)v the kinetic energy (K.E.) and total energy (T.E,) are measured as a function of displacement x. Which of the following statements is true
[1] P.E. is maximum when x = 0
[2] K.E. is maximum when x = 0
[3] T.E. is zero when x = 0
[4] KE. is maximum when x is maximum

25. If <E> and <U> denote the average kinetic and the average potential energies respectively of mass describing a simple harmonic motion, over one period, then the correct relation is
[1] <E> = <U> [2] <E> = 2<U>
[3] <E> = – 2 <U> [4] <E> = – <U>

26. The total energy of a particle, executing simple harmonic motion is
[1] $\propto x$ [2] $\propto x^2$
[3] Independent of x [4] $\propto x^{1/2}$

27. The kinetic energy of a particle executing S.H.M. is 16 J when it is at its mean position. If the mass of the particle is 0.32 kg, then what is the maximum velocity of the particle
[1] 5 m/s [2] 15 m/s
[3] 10 m/s [4] 20 m/s

28. Consider the following statements. The total energy of a particle executing simple harmonic motion depends on its
(a) Amplitude (b) Period
(c) Displacement
Of these statements
[1] (a) and (b) are correct
[2] (b) and (c) are correct
[3] (a) and (c) are correct
[4] (a), (b) and (c) are correct

29. A particle starts simple harmonic motion from the mean position. Its amplitude is a and total energy E. At one instant its kinetic energy is 3E / 4. Its displacement at that instant is
[1] $a/\sqrt{2}$ [2] a/2
[3] $\frac{a}{\sqrt{3/2}}$ [4] $a/\sqrt{3}$

30. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is
[1] $f/2$ [2] f [3] $2f$ [4] $4f$

31. The amplitude of a particle executing SHM is made three – fourth keeping its time period constant. Its total energy will be
[1] E/2 [2] $\frac{3}{4}$E
[3] $\frac{9}{16}$E [4] None of these

32. A particle of mass m is hanging vertically by an ideal spring of force constant K. If the mass is made to oscillate vertically, its total energy is
[1] Maximum at extreme position
[2] Maximum at mean position
[3] Minimum at mean position
[4] Same at all position

33. A body is moving in a room with a velocity of 20 rn / s perpendicular to the two walls separated by 5 meters. There is no friction and the collisions with the walls are elastic. The motion of the body is
[1] Not periodic
[2] Periodic but not simple harmonic
[3] Periodic and simple harmonic
[4] Periodic with variable time period

34. A body is executing Simple Harmonic Motion. At a displacement x its potential energy is E_1 and at a displacement y its potential energy is E_2. The potential energy E at displacement (x + y) is
[1] $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ [2] $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$
[3] $E = E_1 - E_2$ [4] $E = E_1 + E_2$

35. When the kinetic energy of a body executing S.H.M. is 1/3 of the potential energy. The displacement of the body is x percent of the amplitude, where x is
[1] 33 [2] 87 [3] 67 [4] 50

36. Starting from the origin a body oscillates simple harmonically with a period of 2s. After what time will its kinetic energy be 75% of the total energy
[1] 1/4 s [2] 1/3 s
[3] 1/12 s [4] 1/16 s.

37. The total energy of a simple harmonic oscillator is proportional to
[1] Square root of displacement
[2] Velocity
[3] Frequency
[4] Amplitude
[5] Square of the amplitude

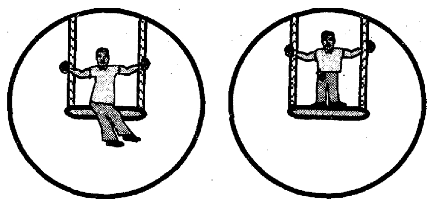
38. The displacement of a particle executing SHM is given by $y = 5 \sin\left(4t + \frac{\pi}{3}\right)$. If T is the time period and mass of the particle is 2g, the kinetic energy of the particle when t = T/4 is given by
[1] 0.4 J [2] 0.5 J
[3] 3 J [4] 0.3 J

39. If a simple pendulum of length L has maximum angular displacement a , then the maximum kinetic energy of bob of mass M is
[1] $\frac{1}{2}\frac{Ml}{g}$ [2] $\frac{Mg}{2L}$
[3] M gL (1 – cos α) [4] MgL sin α/2

Time Period and Frequency

1. A particle moves such that its acceleration a is given by a = –bx, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is
[1] $2\pi\sqrt{b}$ [2] $2\pi/\sqrt{b}$
[3] 2π/b [4] $2\sqrt{\frac{\pi}{b}}$

2. The equation of motion of a particle is $\frac{d^2y}{dt^2} + Ky = 0$, where K is positive constant. The time period of the motion is given by
[1] 2π/K [2] 2πK
[3] $2\pi/\sqrt{K}$ [4] $2\pi\sqrt{K}$

3 

A child swings sitting and standing inside swing as shown in figure, then period of oscillations have the relation
[1] $(T)_{Sitting} = (T)_{Standing}$
[2] $(T)_{Sitting} > (T)_{Standing}$
[3] $(T)_{Sitting} < (T)_{Standing}$
[4] $2(T)_{Sitting} = (T)_{Standing}$

4. The maximum speed of a particle executing S.H.M. is 1m/s and its maximum acceleration is 1.57m/s^2 . The time period of the particle will be

[1] 1/1.57 s [2] 1.57 s
[3] 2 s [4] 4 s.

5. The motion of a particle executing S.H.M. is given by x = 0.01 sin 100π(t + .05), where x is in metres and time is in seconds. The time period is
[1] 0.01s [2] 0.02s
[3] 0.1s [4] 0.2s

6. The kinetic energy of a particle executing S.H.M. is 16 J when it is in its mean position. If the amplitude of oscillations is 25 cm and the mass of the particle is 5.12 kg, the time period of its oscillation is
[1] π/5 s [2] 2π s
[3] 20 π s [4] 5π s

7. The acceleration of a particle performing S.H.M. is 12cm/s^2 at a distance of 3 cm from the mean position. Its time period is
[1] 0.5 s [2] 1.0 s
[3] 2.0 s [4] 3.14 s

8. The displacement x (in metres) of a particle performing simple harmonic motion is related to time t (in seconds) as $x = 0.05\cos\left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be
[1] 0.5 Hz [2] 1.0 Hz
[3] 1.5 Hz [4] 2.0 Hz

9. What is constant in S.H.M.
[1] Restoring force [2] Kinetic energy
[3] Potential energy [4] Periodic time

10. If a simple harmonic oscillator has got a displacement of 0.02 m and acceleration equal to 2.0 ms^{-2} at any time, the angular frequency of the oscillator is equal to
[1] 10 rad s^{-1} [2] 0.1 rad s^{-1}
[3] 100 rad s^{-1} [4] 1 rad s^{-1}

11. A point performs simple harmonic oscillation of period T and the equation of motion is given by x = asin(ω + π/6). After the elapse of what fraction of the time period the velocity of the point will be equal to half its maximum velocity
[1] T/3 [2] T/12 [3] T/8 [4] T/6.

12. Mark the wrong statement
[1] All S.H.M.'s have fixed time period
[2] All motions having same time period are S.H.M.
[3] In S.H.M. total energy is proportional to square of amplitude
[4] Phase constant of S.H.M. depends upon initial conditions

13. A particle in SHM is described by the displacement equation x(t) = Acos(ωt + θ). If the initial (t=0) position of the particle is 1 cm and its initial velocity is π cm/s, what is its amplitude? The angular frequency of the particle is πs^{-1}
[1] 1cm [2] $\sqrt{2}$ cm
[3] 2 cm [4] 2.5 cm

14. A particle executes SHM in a line 4 cm long. Its velocity when passing through the centre of line is 12 cm/s. The period will be
[1] 2.047 s [2] 1.047 s
[3] 3.047 s [4] 0.047 s

15. Two particles A and B execute simple harmonic motion of period T and 5T/4. They start from mean position. The phase difference between them when the particle A complete an oscillation will be
[1] π/2 [2] 0
[3] 2π/5 [4] π/4.

16. A simple harmonic wave having an amplitude a and time period T is represented by the equation y = 5 sin π (t + 4)m. Then the value of amplitude (a) in (m) and time period (T) in second are
[1] a=10, T=2 [2] a=5, T = 1
[3] a=10, T=1 [4] a=5, T = 2

17. A particle executing simple harmonic motion of amplitude 5cm has maximum speed of 31.4 cm/s. The frequency of its oscillation is

[1] 3 Hz [2] 2 Hz
[3] 4 Hz [4] 1 Hz

18. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium it undergoes oscillation with a time period T. Then

[1] $T \propto \frac{1}{\rho}$ [2] $T \propto \frac{1}{\sqrt{m}}$

[3] $T \propto \sqrt{\rho}$ [4] $T \propto \frac{1}{\sqrt{A}}$

19. What is the effect on the time period of a simple pendulum if the mass of the bob is doubled

[1] Halved [2] Doubled
[3]Becomes eight times [4] No effect

20. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. The minimum time taken by the particle to travel half of the amplitude from the equilibrium position is

[1] T/4 [2] T/8
[3] T/12 [4] T/2

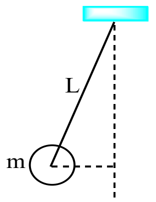
21. The equation of a damped simple harmonic motion is $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$. Then the angular frequency of

[1] $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)^{1/2}$ [2] $\omega = \left(\frac{k}{m} - \frac{b}{4m}\right)^{1/2}$

[3] $\omega = \left(\frac{k}{m} - \frac{b^2}{4m}\right)^{1/2}$ [4] $\omega = \left(\frac{k}{m} - \frac{b^2}{4m^2}\right)$

22. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

[1] 9
[2] 18
[3] 27
[4] 36



23. A particle executes SHM with amplitude 0.2 m and time period 24 s. The time required for it to move from the mean position to a point 0.1 m is

[1] 2 s [2]3 s [3] 8s [4] 12 s

24. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then

[1] Amplitude of motion is 3a
[2] Time period of oscillations is 8 τ
[3] Amplitude of motion is 4 τ
[4] Time period of oscillations is 6 τ

Simple Pendulum

1. The period of a simple pendulum is doubled, when

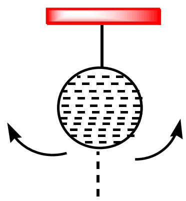
[1] Its length is doubled
[2] The mass of the bob is doubled
[3] Its length is made four times
[4]The mass of the bob and the length of the pendulum are doubled

2. The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is

[1] Greater than T [2]Less than T
[3] Equal to T
[4]Cannot be compared

3. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will

[1] Remains unchanged
[2] Increase
[3] Decrease
[4] Become erratic



4. How does the time period of pendulum vary with length

[1] $\sqrt{L}$ [2] $\sqrt{\frac{L}{2}}$
[3] $1/\sqrt{L}$ [4] 2L

5. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be (If it is a second's pendulum on earth)

[1] $1/\sqrt{2}$ s [2] $2\sqrt{2}$ s
[3] 2 s [4] 1/2 s.

6. A simple pendulum is set up in a trolley which moves to the right with an acceleration a on a horizontal plane. Then the thread of the pendulum in the mean position makes an angle θ with the vertical

[1] $\tan^{-1}\frac{a}{g}$ in the forward direction

[2] $\tan^{-1}\frac{a}{g}$ in the backward direction

[3] $\tan^{-1}\frac{a}{g}$ in the backward direction

[4] $\tan^{-1}\frac{a}{g}$ in the forward direction

7. A simple pendulum has a time period T in vacuum. Its time period when it is completely immersed in a liquid of density one-eight of the density of material of the bob is

[1] $\sqrt{\frac{7}{8}}\,T$ [2] $\sqrt{\frac{5}{8}}\,T$

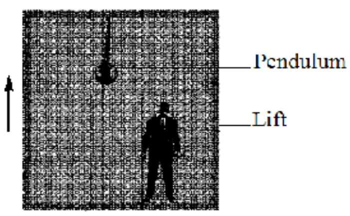
[3] $\sqrt{\frac{3}{8}}\,T$ [4] $\sqrt{\frac{8}{7}}\,T$

8. A pendulum of length 1 m is released from $\theta = 60°$. The rate of change of speed of the bob at $\theta = 30°$ is ($g = 10ms^{-2}$)

[1] $10ms^{-2}$ [2] $7.5\ ms^{-2}$
[3] $5\ ms^{-2}$ [4] $5\sqrt{3}\ ms^{-2}$

9. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T s. If the lift accelerates upwards with an acceleration g/4, then the period of the pendulum will be

[1] T
[2] T/4
[3] $2T/\sqrt{5}$
[4] $2T\sqrt{5}$



10. A simple pendulum is suspended from the roof of a trolley which moves in a horizontal direction with an acceleration a, then the time period is given by $T = 2\pi\sqrt{\frac{l}{g'}}$, where g is equal to

[1] g [2] g – a
[3] g + a [4] $\sqrt{g^2 + a^2}$

11. A second's pendulum is placed in a space laboratory orbiting around the earth at a height 3R, where R is the radius of the earth. The time period of the pendulum is

[1] Zero [2] $2\sqrt{3}$ s
[3] 4 s [4] Infinite

12. Two pendulum of lengths 1.44 m and 1 m start to swing together. The number of vibrations after which they will again start swinging together is

[1] 4 [2] 3 [3]6
[4] 2 [5] 5

13. Choose the correct statement

[1]Time period of a simple pendulum depends on amplitude.
[2] Time shown by a spring watch varies with acceleration due to gravity g.
[3] In a simple pendulum time period varies linearly with the length of the pendulum
[4] The graph between length of the pendulum and time period is a parabola.

14. If the length of second's pendulum is decreased by 2%, how many seconds it will lose per day

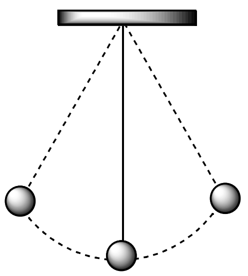
[1] 3927 s [2] 3727 s
[3] 3427 s [4] 864 s

15. A simple pendulum has time period T. The bob is given negative charge and surface below it is given positive charge. The new time period will be

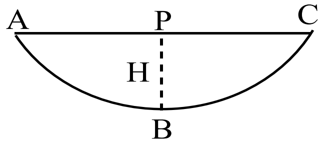
[1] Less than T [2] Greater than T
[3] Equal to T [4] Infinite

16. What is the velocity of the bob of a simple pendulum at Its mean position, if it is able to rise to vertical height of 10cm ($g = 9.8\ m/s^2$)

[1] 2.2m/s
[2] 1.8m/s
[3] 1.4 m/s
[4] 0.6m/s



17. A simple pendulum with a bob of mass 'm' oscillates from A to C and back to A such that PB is H. If the acceleration due to gravity is 'g', then the velocity of the bob as it passes through B is



[1] mgH
[2] $\sqrt{2gH}$
[3] 2gH
[4] Zero

18. Which one of the following is simple harmonic
[1] Rotation of earth around the sun
[2] Rotation of earth about its own axis
[3] Revolving motion of a top
[4] Motion of a steel ball in a viscous medium
[5] Motion of oscillating liquid column in U tube

19. The length of the simple pendulum which ticks seconds is
[1]2m [2] 1.5m [3] 1 m
[4] 3 m [5] 0.5 m

20. If a body is released into a tunnel dug across the diameter of earth, it executes simple harmonic motion with time period
[1] $T = 2\pi\sqrt{\frac{R_e}{g}}$ [2] $T = 2\pi\sqrt{\frac{2R_e}{g}}$
[3] $T = 2\pi\sqrt{\frac{R_e}{2g}}$ [4] T = 2 seconds

21. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0 s to t = τ5, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
[1] 0.693/ b [2] h
[3]1/b [4] 2/b

22. The time period of a simple pendulum is 2 s. If its length is increased 4 times, then its period becomes
[1] 16 s [2]12 s [3] 8 s [4] 4 s

23. The time period of a simple pendulum of length L as measured in an elevator descending with acceleration g/3 is
[1] $2\pi\sqrt{\frac{3L}{g}}$ [2] $\pi\sqrt{\left(\frac{3L}{g}\right)}$
[3] $2\pi\sqrt{\left(\frac{3L}{2g}\right)}$ [4] $2\pi\sqrt{\frac{2L}{3g}}$

24. A pendulum is made to hang from the ceiling of an elevator. It has period of T s (for small angles). The elevator is made to accelerate upwards with $10 m/s^2$. The period of the pendulum now will be (assume $g = 10 m/s^2$)
[1] $T\sqrt{2}$ [2] Infinite
[3] $T/\sqrt{2}$ [4] Zero

25. A pendulum bob of mass m is hanging from a fixed point by a light thread of length l. A horizontal speed v_0 is imparted to the bob so that it takes up horizontal position. If g is the acceleration due to gravity, then v0 is
[1] mgl [2] $\sqrt{2gl}$
[3] $\sqrt{gl}$ [4] gl

26. A simple pendulum is executing simple harmonic motion with a time period T. If the length of the pendulum is increased by 21%, the percentage increase in the time period of the pendulum of increased length is
[1] 10% [2] 21%
[3] 30% [4] 50%

27. A simple pendulum is vibrating in an evacuated chamber, it will oscillate with
[1] Increasing amplitude
[2] Constant amplitude
[3] Decreasing amplitud
[4] First (3) then (1)

28. The velocity of simple pendulum is maximum at
[1] Extremes [2] Half displacement
[3] Mean position [4] Every where

29. The time period of a simple pendulum in a lift descending with constant acceleration g is
[1] $T = 2\pi\sqrt{l/g}$ [2] $T = 2\pi\sqrt{l/2g}$
[3] Zero [4] Infinite.

30. A chimpanzee swinging on a swing in a sitting position, stands up suddenly, the time period will
[1] Become infinite [2] Remain same
[3] Increase [4] Decrease

31. The acceleration due to gravity at a place is π^2 m/s^2. Then the time period of a simple pendulum of length one metre is
[1] $2/\pi$ s [2] 2πs
[3] 2 s [4] π s.

32. A plate oscillates with time period 'T'. Suddenly, another plate put on the first plate, then time period
[1] Will decrease [2] Will increase
[3] Will be same [4] None of these

33. A simple pendulum of length l has a brass bob attached at its lower end. Its period is T. If a steel bob of same size, having density x times that of brass, replaces the brass bob and its length is changed so that period becomes 2T, then new length is
[1] 2 l [2] 4 l [3] 4 l x [4] $4l/x$

34. In a seconds pendulum, mass of bob is 30 g. If it is replaced by 90 gm mass. Then its time period will
[1] 1 s [2] 2 s [3] 4 s [4] 3 s

35. The time period of a simple pendulum when it is made to oscillate on the surface of moon
[1] Increases [2] Decreases
[3] Remains unchanged [4] Becomes infinite

36. A simple pendulum is attached to the roof of a lift. If time period of oscillation, when the lift is stationary is T. Then frequency of oscillation, when the lift falls freely, will be
[1] Zero [2] T
[3] 1/T [4] None of these

37. A simple pendulum, suspended from the ceiling of a stationary van, has time period T. If the van starts moving with a uniform velocity the period of the pendulum will be
[1] Less than T [2] Equal to 2T
[3] Greater than T [4] Unchanged

38. Length of a simple pendulum is l and its maximum angular displacement is θ, then its maximum K.E. is
[1] $mgl \sin\theta$ [2] $mgl(1 + \sin\theta)$
[3] $mgl(1 + \cos\theta)$ [4] $mgl(1 - \cos\theta)$

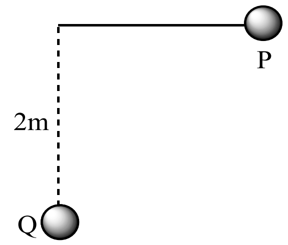
39. To show that a simple pendulum executes simple harmonic motion, it is necessary to assume that
[1] Length of the pendulum is small
[2] Mass of the pendulum is small
[3] Amplitude of oscillation is small
[4] Acceleration due to gravity is small

40. The height of a swing changes during its motion from 0.1 m to 2.5 m. The minimum velocity of a boy who swings in this swing is
[1] 5.4 m/s [2] 4.95 m/s
[3] 3.14 m/s [4] Zero

41. The amplitude of an oscillating simple pendulum is 10 cm and its period is 4 s. Its speed after 1 s after it passes its equilibrium position, is
[1] Zero [2] 0.57 m/s
[3] 0.212 m/s [4] 0.32 m/s

42. There is a simple pendulum hanging from the ceiling of a lift. When the lift is stand still, the time period of the pendulum is T. If the resultant acceleration becomes g/4, then the new time period of the pendulum is
[1] 0.8 T [2] 0.25 T
[3] 2 T [4] 4 T

43. A pendulum of length 2m lift at P. When it reaches Q, it losses 10% of its total energy due to air resistance. The velocity at Q is
[1] 6 m/s
[2] 1 m/s
[3] 2 m/s
[4] 8 m/s



44. A simple pendulum is suspended from the ceiling of a stationary elevator and its period of oscillation is T. The elevator is then set into motion and the new time period is found to be longer. Then the elevator is
[1] Accelerated upward
[2] Accelerated downward
[3] Moving downward with nonuniform speed
[4] Moving downward with uniform speed

45. The ratio of frequencies of two pendulums are 2 :3 , then their length are in ratio
[1] $\sqrt{2/3}$ [2] $\sqrt{3/2}$
[3] 4/9 [4] 9/4

46. A simple pendulum is taken from the equator to the pole. Its period
[1] Decreases [2] Increases
[3] Remains the same
[4] Decreases and then increases

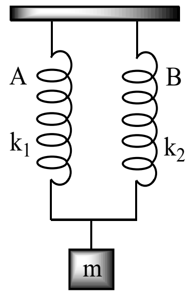
47. A simple pendulum hanging from the ceiling of a stationary lift has a time period T_1. When the lift moves downward with constant velocity, the time period is T_2, then
[1] T_2 is infinity [2] $T_2 > T_1$
[3] $T_2 < T_1$ [4] $T_2 = T_1$

48. If the length of a pendulum is made 9 times and mass of the bob is made 4 times then the value of time period becomes
[1] 3 T [2] 3/2T
[3] 4T [4] 2T.

SPRING PENDULUM

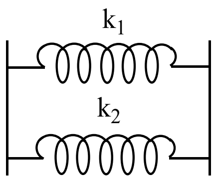
1. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is
[1] k_1/k_2 [2] $\sqrt{k_1/k_2}$
[3] k_2/k_1 [4] $\sqrt{k_2/k_1}$

2. A mass m is suspended by means of two coiled spring which have the same length in unstretched condition as in figure. Their force constant are k_1 and k_2 respectively. When set into vertical vibrations, the period will be
[1] $2\pi\sqrt{\left(\frac{m}{k_1 k_2}\right)}$
[2] $2\pi\sqrt{m\left(\frac{k_1}{k_2}\right)}$
[3] $2\pi\sqrt{\left(\frac{m}{k_1 - k_2}\right)}$
[4] $2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

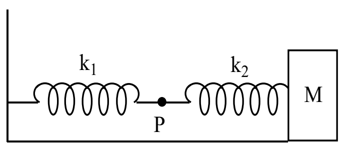


3. A spring has a certain mass suspended from it and its period for vertical oscillation is T. The spring is now cut into two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillation is now
[1] T/2 [2] $T/\sqrt{2}$
[3] $\sqrt{2}$ T [4] 2T.

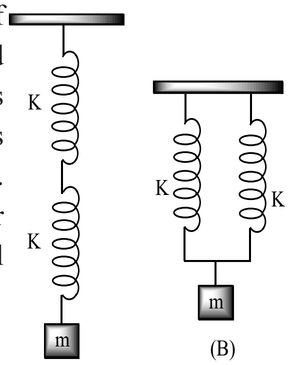
4. Two springs of spring constants k_1 and k_2 are connected as shown. The effective spring constant k_e =
[1] $k_1 + k_2$
[2] $k_1k_2/k_1 + k_2$
[3] k_1k_2
[4] k_1/k_2



5. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is
[1] k_1A/k_2
[2] k_2A/k_1
[3] $k_1A/k_1 + k_2$
[4] $k_2A/k_1 + k_2$



6. Two identical springs of constant K are connected in series and parallel as shown in figure. A mass m is suspended from them. The ratio of their frequencies of vertical oscillations will be
[1] 2:1 [2] 1:1
[3] 1:2 [4] 4:1



(A) (B)

7. A mass m is suspended from the two coupled springs connected in series. The force constant for springs are K_1 and K_2. The time period of the suspended mass will be

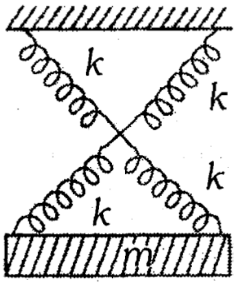
[1] $T = 2\pi\sqrt{\left(\frac{m}{K_1 + K_2}\right)}$

[2] $T = 2\pi\sqrt{\left(\frac{2m}{K_1 + K_2}\right)}$

[3] $T = 2\pi\sqrt{\left(\frac{m(K_1 + K_2)}{K_1 K_2}\right)}$

[4] $T = 2\pi\sqrt{\left(\frac{mK_1 K_2}{K_1 + K_2}\right)}$

8. As shown in figure, a simple harmonic motion oscillator having identical four springs has time period



[1] $T = 2\pi\sqrt{\frac{m}{4k}}$

[2] $T = 2\pi\sqrt{\frac{m}{2k}}$

[3] $T = 2\pi\sqrt{\frac{m}{k}}$

[4] $T = 2\pi\sqrt{\frac{2m}{k}}$

9. A block of mass 500 g is connected to a spring of spring constant k = 312.5 N/m on a frictionless table. The spring is held firmly at the other end. The block is pulled a distance of 5 cm and then released to make SHM. Calculate the time period of its oscillations

[1] 2.0 s [2] 1.75 s
[3] 0.5 s [4] 0.25 s

10. A spring having a spring constant 'K' is loaded with a mass 'm'. The spring is cut into two equal parts and one of these is loaded again with the same mass. The new spring constant is

[1] K/2 [2] K [3]2K [4] K^2

11. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency

[1] $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ [2] $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

[3] $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ [4] $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

12. A mass M is suspended from a light spring. An additional mass m added displaces the spring further by a distance x. Now the combined mass will oscillate on the spring with period

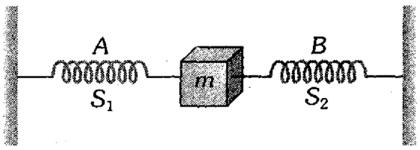
[1] $T = 2\pi\sqrt{(mg/x(M+m))}$

[2] $T = 2\pi\sqrt{((M+m)x/mg)}$

[3] $T = (\pi/2)\sqrt{(mg/x(M+m))}$

[4] $T = 2\pi\sqrt{((M+m)/mgx)}$

13. In the figure, S_1 and S_2 are identical springs. The oscillation frequency of the mass m is *f* If one spring is removed, the frequency will become



[1] f
[2] f x 2
[3] f x $\sqrt{2}$
[4] f/ $\sqrt{2}$

14. The vertical extension in a light spring by a weight of 1 kg suspended from the wire is 9.8 cm. The period of oscillation

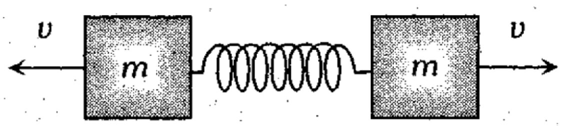
[1] 20 πs [2] 2 π s
[3] 2π/10 s [4] 200 π s.

15. A small mass m attached to one end of a spring with a negligible mass and an unstretched length L, executes vertical oscillations with angular frequency ω_0. When the mass is rotated with an angular speed ω by holding the other end of the spring at a fixed point, the mass moves uniformly in a circular path in a horizontal plane. Then the increase in length of the spring during this rotation is

[1] $\frac{\omega^2 L}{\omega_0^2 - \omega^2}$ [2] $\frac{\omega_0^2 L}{\omega - \omega_0^2}$

[3] $\frac{\omega^2 L}{\omega_0^2}$ [4] $\frac{\omega^2 L}{\omega^2}$

16. Two blocks each of mass m are connected to a spring of spring constant k. If both are given velocity v in opposite directions, then the maximum elongation of the spring is



[1] $\sqrt{\frac{mv^2}{k}}$ [2] $\sqrt{\frac{2mv^2}{k}}$

[3] $\sqrt{\frac{mv^2}{2k}}$ [4] $2\sqrt{\frac{mv^2}{k}}$

17. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2 . The ratio of (A_1/A_2) is

[1] M/M+m [2] M+m/M

[3] $\left(\frac{M}{M+m}\right)^{1/2}$ [4] $\left(\frac{M+m}{m}\right)^{1/2}$

18. A spring executes SHM with mass of 10 kg attached to it. The force constant of spring is 10 N/m. If at any instant its velocity is 40cm/s, the displacement will be (where amplitude is 0.5m)

[1] 0.09 m [2] 0.3 m

[3] 0.03 m [4] 0.9 m

19. Time period of a mass m suspended by a spring is T. If the spring is cut to one-half and made to oscillate by suspending double mass, the time period of the mass will be

[1] 8T [2] T

[3] T/2 [4] T

20. A block of mass m, attached to a spring of spring constant *k*, oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. The block has a speed v when the spring is at its natural length. Before coming to an instantaneous rest, if the block moves a distance x from the mean position, then

[1] $x = \sqrt{m/k}$ [2] $x = 1/v\sqrt{m/k}$

[3] $x = v\sqrt{m/k}$ [4] $x = \sqrt{mv/k}$

21. The force constants of two springs are K_1 and K_2. Both are stretched till their elastic energies are equal. If the stretching forces are F_1 and F_2, then $F_1:F_2$ is

[1] $K_1:K_2$ [2] $K_2:K_1$

[1] $\sqrt{K_1}:\sqrt{K_2}$ [4] $K_1^2:K_2^2$

22. A mass m is vertically suspended from a spring of negligible mass; the system oscillates with a frequency n. What will be the frequency of the system if a mass 4 m is suspended from the same spring

[1] n/4 [2] 4n

[3] n/2 [4] 2n

23. If the period of oscillation of mass m suspended from a spring is 2 s, then the period of mass 4m will be

[1] 1 s [2] 2s

[3] 3 s [4] 4 s

24. If two springs A and B with spring constants 2k and k, are stretched separately by same suspended weight, then the ratio between the work done in stretching A and B is

[1] 1 :2 [2]1 :4 [3] 1 :3

[4] 4 :1 [4] 2:1

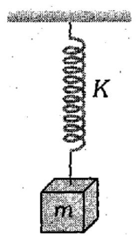
25. A mass m performs oscillations of period T when hanged by spring of force constant K. If spring is cut in two parts and arranged in parallel and same mass is oscillated by them, then the new time period will be

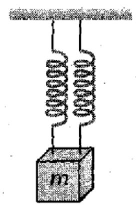
[1] 2T

[2] T

[3] $T/\sqrt{2}$

[4] T/2



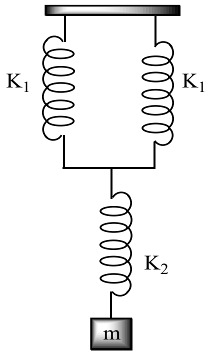


26. A body of mass 4.9 kg hangs from a spring and oscillates with a period 0.5 s. On the removal of the body, the spring is shortened by (Take $g = 10\ ms^{-2}$, $\pi^2 = 10$)

[1] 6.3 m [2] 0.63 m [3] 6.25 cm

[4] 63 cm [5] 0.625 cm

27. What will be the force constant of the spring system shown in the figure



[1] $\frac{K_1}{2}+K_2$

[2] $\left[\frac{1}{2K_1}+\frac{1}{K_2}\right]^{-1}$

[3] $\frac{1}{2K_1}+\frac{1}{K_2}$

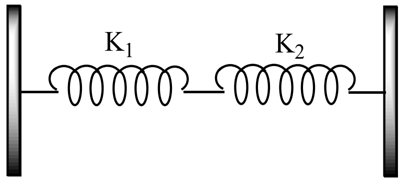
[4] $\left[\frac{2}{K_1}+\frac{1}{K_2}\right]^{-1}$

28. Two springs have spring constants K_A and K_B and $K_A > K_B$. The work required to stretch them by same extension will be

[1] More in spring A [2] More in spring B

[3] Equal in both [4] Noting can be said

29. The effective spring constant of two spring system as shown in figure will be



[1] $K_1 + K_2$ [2] $K_1K_2/ K_1 + K_2$

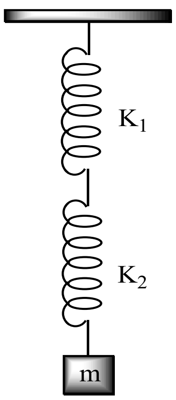
[3] $K_1 - K_2$ [4] $K_1K_2/ K_1 - K_2$

30. A mass m attached to a spring oscillates every 2 s. If the mass is increased by 2 kg, then time-period increases by 1 s. The initial mass is

[1] 1.6 kg [2] 3.9 kg

[3] 9.6 kg [4] 12.6 kg

31. A mass M is suspended by two springs of force constants K_1 and K_2 respectively as shown in the diagram. The total elongation (stretch) of the two springs is



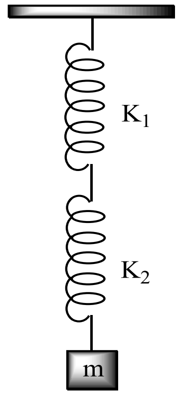
[1] $\frac{Mg}{K_1+K_2}$

[2] $\frac{Mg(K_1+K_2)}{K_1K_2}$

[3] $\frac{MgK_1K_2}{K_1+K_2}$

[4] $\frac{K_1+K_2}{K_1K_2Mg}$

32. The frequency of oscillation of the springs shown in the figure will be



[1] $\frac{1}{2\pi}\sqrt{\frac{K}{m}}$

[2] $\frac{1}{2\pi}\sqrt{\frac{(K_1+K_2)m}{K_1K_2}}$

[3] $2\pi\sqrt{\frac{K}{m}}$

[4] $\frac{1}{2\pi}\sqrt{\frac{K_1K_2}{(K_1+K_2)m}}$

33. The scale of a spring balance reading from 0 to 10 kg is 0.25 m long. A body suspended from the balance oscillates vertically with a period of k/10 second. The mass suspended is (neglect the mass of the spring)

[1] 10kg [2] 0.98 kg

[3] 5 kg [4] 20 kg

34. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be

[1] $T\sqrt{n}$ [2] $T/\sqrt{n}$

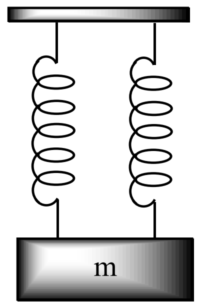
[3] nT [4] T.

35. One-forth length of a spring of force constant K is cut away. The force constant of the remaining spring will be

[1] 3/4 K [2] 4/3 K

[3] K [4] 4 K.

36. A mass m is suspended separately by two different springs of spring constant K_1 and K_2 gives the time-period t_1 and t_2 respectively. If same mass m is connected by both springs as shown in figure then time-period t is given by the relation



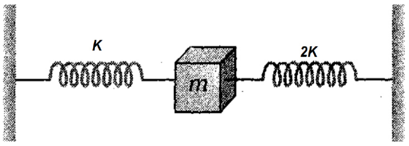
[1] $t = t_1 + t_2$

[2] $t = t_1 t_2/ t_1 + t_2$

[3] $t^2 = t_1^{\,2} + t_2^{\,2}$

[4] $t^{-2} = t_1^{-2} + t_2^{-2}$

37. Two springs of force constants K and 2K are connected to a mass as shown below. The frequency of oscillation of the mass is



[1] $(1/2\pi)\sqrt{(K/m)}$ [2] $(1/2\pi)\sqrt{(2K/m)}$
[3] $(1/2\pi)\sqrt{(3K/m)}$ [4] $(1/2\pi)\sqrt{(m/K)}$

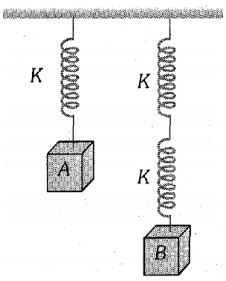
38. Two springs of constant k_1 and k_2 are joined in series. The effective spring constant of the combination is given by
[1] $\sqrt{k_1k_2}$ [2] $(k_1+k_2)/2$
[3] k_1+k_2 [4] $k_1k_2/(k_1+k_2)$

39. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
[1] $T = t_1 + t_2$ [2] $T^2 = t_1^2 + t_2^2$
[3] $T^{-1} = t_1^{-1} + t_2^{-1}$ [4] $T^{-2} = t_1^{-2} + t_2^{-2}$

40. Infinite springs with force constant k, 2k, 4k and 8k... respectively are connected in series. The effective force constant of the spring will be
[1] 2k [2] k [3] k/2 [4] 2048

41. To make the frequency double of a spring oscillator, we have to
[1] Reduce the mass to one fourth
[2] Quadruple the mass
[3] Double of mass [4] Half of the mass

42. The springs shown are identical. When A = 4kg, the elongation of spring is 1 cm. If B = 6kg, the elongation produced by it is
[1] 4 cm
[2] 3 cm
[3] 2 cm
[4] 1cm



43. When a body of mass 1.0 kg is suspended from a certain light spring hanging vertically, its length increases by 5 cm. By suspending 2.0 kg block to the spring and if the block is pulled through 10 cm and released the maximum velocity in it in m/s is (Acceleration due to gravity = $10 m/s^2$)
[1] 0.5 [2] 1
[3] 2 [4] 4

44. Two springs with spring constants K_1 =1500 N/m and K_2 = 3000 N/m are stretched by the same force. The ratio of potential energy stored in spring will be
[1] 2:1 [2] 1:2 [3] 4 :1 [4] 1 :4

45. If a spring extends by x on loading, then energy stored by the spring is (if T is the tension in the spring and K is the spring constant)
[1] $T^2/2x$ [2] $T^2/2K$
[3] $2K/T^2$ [4] $2T^2/K$.

46. A weightless spring of length 60 cm and force constant 200 N/m is kept straight and unstretched on a smooth horizontal table and its ends are rigidly fixed. A mass of 0.25 kg is attached at the middle of the spring and is slightly displaced along the length. The time period of the oscillation of the mass is
[1] $\frac{\pi}{20}$ s [2] $\frac{\pi}{10}$ s
[3] $\frac{\pi}{5}$ s [4] $\frac{\pi}{\sqrt{200}}$ s

47. The time period of a mass suspended from a spring is T. If the spring is cut into four equal parts and the same mass is suspended from one of the parts, then the new time period will be
[1] T [2] T/2 [3] 2 7 [4]7/4

48. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes S.H.M. of time period T. If the mass is increased by m, the time period becomes 5T/3. Then the ratio of m/M is
[1] 5/3 [2] 3/5
[3] 25/9 [4] 16/9

49. An object is attached to the bottom of a light vertical spring and set vibrating. The maximum speed of the object is 15 cm/s and the period is 628 milliseconds. The amplitude of the motion in centimeters is

[1] 3.0 [2] 2.0 [3] 1.5 [4] 1.0

50. When a mass m is attached to a spring, it normally extends by 0.2 m. The mass m is given a slight addition extension and released, then its time period will be

[1] 1/7 sec [2] 1 sec
[3] $2\pi/7$ sec [4] $2/3\pi$ sec

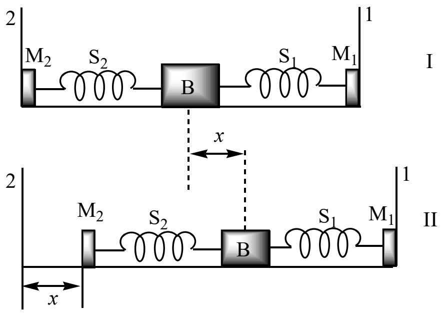
51. If a body of mass 0.98 kg is made to oscillate on a spring of force constant 4.84 N/m, the angular frequency of the body is

[1] 1.22rad/s [2] 2.22 rad/s
[3] 3.22 rad/s [4] 4.22 rad/s

52. A mass m is suspended from a spring of length l and force constant K. The frequency of vibration of the mass is f_1. The spring is cut into two equal parts and the same mass is suspended from one of the parts. The new frequency of vibration of mass is f_2. Which of the following relations between the frequencies is correct

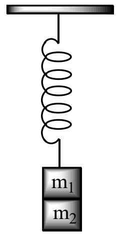
[1] $f_1 = \sqrt{2} f_2$ [2] $f_1 = f_2$
[3] $f_1 = 2f_2$ [4] $f_2 = \sqrt{2} f_1$

53. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is



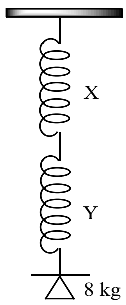
[1] 4
[2] 2
[3] 1/2
[4] 1/4

54. Two masses m_1 and m_2 are suspended together by a massless spring of constant K. When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillations is



[1] m_1g/K
[2] m_1g/K
[3] $(m_1 + m_2)/K$
[4] $(m_1 - m_2)g/K$

55. A body of mass 8 kg is suspended through two light springs X and V connected in series as shown in figure. The readings in X and Y respectively are



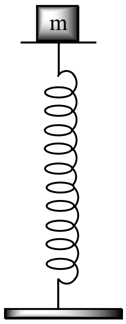
[1] 8 kg, zero
[2] Zero, 8 kg
[3] 8 kg, 8 kg
[4] 2 kg, 6 kg

56. A body of mass 20 g connected to spring of constant K executes simple harmonic motion with a frequency of $(5/\pi)$Hz . The value of spring constant is

[1] 4 Nm^{-1} [2] 3 Nm^{-1}
[3] 2 Nm^{-1} [4] 5 Nm^{-1}

57. A mass of 2.0 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible. When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N/m.

What should be the minimum amplitude of the motion so that the mass gets detached from the pan (Take g = 10m/s^2)



[1] 8.0cm
[2] 10.0cm
[3]Any value less than 12.0 cm
[4] 4.0 cm

58. A mass of 10 kg is suspended from a spring balance. It is pulled aside by a horizontal string so that it makes an angle of 60° with the vertical. The new reading of the balance is
[1] 10 $\sqrt{3}$ kgwt [2] 20 $\sqrt{3}$ kg wt
[3] 20 kg wt [4] 10 kg wt

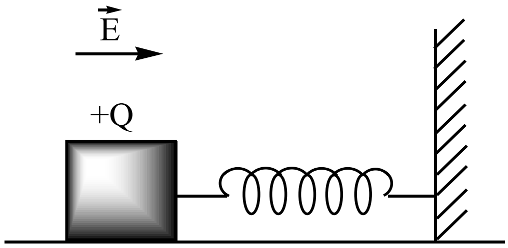
59. An electric motor of mass 40 kg is mounted on four vertical springs each having constant of 4000 Nm^{-1}. The period with which the motor vibrates vertically is
[1] 0.314 s [2] 3.14 s
[3] 0.628 s [4] 0.56 s
[5] 0.078 s

60. An ideal spring with spring constant K = 200 N/m is fixed on one end on a wall. If the spring is pulled with a force 10 N at the other end along its length, how much it will extended?
[1] 5 cm [2] 2 m
[3] 2 cm [4] 5 m

61. A mass of 4 kg suspended from a spring of force constant 800 Nm^{-1} executes simple harmonic oscillations. If the total energy of the oscillator is 4J, the maximum acceleration (in ms^{-2}) of the mass is
[1] 5 [2] 15 [3] 45 [4] 20 [5]25

62. A body of mass 500 g is attached to a horizontal spring of spring constant 8π^2N m^{-1}. If the body is pulled to a distance of 10 cm from its mean position, then its frequency of oscillation is
[1] 2 Hz [2]4 Hz [3] 8 Hz
[4]0.5 Hz [5] 4π Hz

63. A wooden block performs SHM on a frictionless surface with frequency, v_0, The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then SHM of the block will be



[1] Of the same frequency and with shifted mean position
[2] Of the same frequency and with the same mean position
[3] Of changed frequency and with shifted mean position
[4] Of changed frequency and with the same mean position

Superposition of S.H.M's and Resonance

1. Two simple harmonic motions are represented by
$y_1 = 5 (\sin 2\pi t + \sqrt{3} \cos 2\pi t)$
$y_2 = 5 \sin \left(2\pi t + \frac{\pi}{4}\right)$
The ratio of the amplitudes of two SHM's is
[1] 1:1 [2] 1:2
[3] 2:1 [4] 1:$\sqrt{3}$

2. In damped oscillations, the amplitude of oscillations is reduced to one-third of its initial value a_0 at the end of 100 oscillations. When the oscillator completes 200 oscillations, its amplitude must be
[1] $a_0/2$ [2] $a_0/4$
[3] $a_0/6$ [3] $a_0/9$.

3. The motion of a particle varies with time according to the relation $y = a(\sin \omega t + \cos \omega t)$, then
[1] The motion is oscillatory but not S.H.M.
[2] The motion is S.H.M. with amplitude a

[3] The motion is S.H.M. with amplitude $a\sqrt{2}$
[4] The motion is S.H.M. with amplitude 2a

4. The resultant of two rectangular simple harmonic motions of the same frequency and unequal amplitudes but differing in phase by $\pi/2$ is
[1] Simple harmonic [2] Circular
[3] Elliptical [4] Parabolic

5. The composition of two simple harmonic motions of equal periods at right angle to each other and with a phase difference of π results in the displacement of the particle along
[1] Straight line [2] Circle
[3] Ellipse [4] Figure of eight

6. A simple pendulum oscillates in air with time period T and amplitude A. As the time passes
[1] T and A both decrease
[2] T increases and A is constant
[3] T remains same and A decreases
[4] T decreases and A is constant

7. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is
[1] 8 [2] –4
[3] 4 [4] $4\sqrt{2}$

8. A S.H.M. is represented by $x = 5\sqrt{2}$ (sin $2\pi t$ + cos $2\pi t$).The amplitude of the S.H.M. is
[1] 10 cm [2] 20 cm
[3] $5\sqrt{2}$ cm [4] 50 cm

9. Resonance is an example of
[1] Tuning fork [2] Forced vibration
[3] Free vibration [4] Damped vibration

10. In case of a forced vibration, the resonance wave becomes very sharp when the
[1] Restoring force is small
[2] Applied periodic force is small
[3] Quality factor is small
[4] Damping force is-small

11. Two simple harmonic mo&ons are represented by $y_1 = 4\sin(4\pi t + \pi/2)$ and $y_2 = 3\cos(4\pi t)$. The resultant amplitude is
[1] 7 [2]1 [3] 5
[4] $2 + \sqrt{3}$ [5] $2 - \sqrt{3}$

12. A particle with restoring force proportional to displacement and resisting force proportional to velocity is subjected to a force F sin ωt. If the amplitude of the particle is maximum for $\omega = \omega_1$ and the energy of the particle is maximum for $\omega = \omega_2$, then (where ω_0 natural frequency of oscillation of particle)
[1] $\omega_1 = \omega_0$ and $\omega_2 \neq \omega_0$
[1] $\omega_1 = \omega_0$ and $\omega_2 = \omega_0$
[1] $\omega_1 \neq \omega_0$ and $\omega_2 = \omega_0$
[1] $\omega_1 \neq \omega_0$ and $\omega_2 \neq \omega_0$

13. A simple pendulum is set into vibrations. The bob of the pendulum comes to rest after some time due to
[1] Air friction
[2] Moment of inertia
[3] Weight of the bob
[4] Combination of all the abovi

Exercise – 1

Displacement of S.H.M. and Phase

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**4**	**3**	**4**	**3**	**2**	**3**	**1**	**1**	**4**	**1**	**3**	**3**	**3**	**3**	**4**
Q	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	**3**	**4**	**1**	**1**	**1**	**2**	**3**	**1**	**3**	**1**	**1**	**1**	**2**	**4**	**2**

31 – 3

Velocity of Simple Harmonic Motion

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**1**	**3**	**1**	**3**	**2**	**1**	**4**	**3**	**2**	**2**	**1**	**4**	**4**	**3**	**3**
Q	16	17	18	19	20	21	22	23							
A	**2**	**1**	**1**	**4**	**2**	**2**	**3**	**3**							

Acceleration of simple Harmonic Motion

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**4**	**2**	**3**	**2**	**1**	**4**	**1**	**4**	**4**	**4**	**1**	**1**	**4**	**1**	**1**
Q	16	17	18	19	20	21	22	23	24						
A	**3**	**4**	**4**	**2**	**3**	**3**	**4**	**2**	**3**						

Energy of Simple Harmonic Motion

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**4**	**1**	**4**	**3**	**3**	**3**	**3**	**2**	**4**	**3**	**4**	**2**	**1**	**2, 4**	**2**
Q	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	**2**	**3**	**2**	**4**	**3**	**3**	**2**	**2**	**2**	**1**	**3**	**3**	**1**	**2**	**3**
Q	31	32	33	34	35	36	37	38	39						
A	**3**	**4**	**2**	**2**	**2**	**4**	**5**	**4**	**3**						

Time period and Frequency

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**2**	**3**	**2**	**4**	**2**	**1**	**4**	**4**	**4**	**1**	**2**	**2**	**2**	**2**	**3**
Q	16	17	18	19	20	21	22	23	24						
A	**4**	**4**	**4**	**4**	**3**	**1**	**4**	**1**	**4**						

Simple Pendulum

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**3**	**1**	**2**	**1**	**2**	**2**	**4**	**3**	**3**	**4**	**4**	**5**	**4**	**4**	**1**
Q	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	**3**	**2**	**5**	**3**	**1**	**4**	**4**	**3**	**3**	**2**	**1**	**2**	**3**	**4**	**4**
Q	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A	**3**	**3**	**2**	**2**	**1**	**1**	**4**	**4**	**3**	**4**	**1**	**3**	**1**	**2**	**4**
Q	46	47	48												
A	**1**	**4**	**1**												

Spring Pendulum

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	**4**	**4**	**2**	**1**	**4**	**3**	**3**	**3**	**4**	**3**	**3**	**2**	**4**	**3**	**4**
Q	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
A	**2**	**4**	**2**	**4**	**3**	**3**	**3**	**4**	**1**	**4**	**3**	**2**	**1**	**1**	**1**
Q	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
A	**2**	**4**	**2**	**2**	**2**	**4**	**3**	**4**	**2**	**3**	**1**	**2**	**2**	**1**	**2**
Q	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	**1**	**2**	**4**	**3**	**3**	**2**	**4**	**3**	**1**	**3**	**3**	**2**	**3**	**1**	**1**
Q	61	62	63												
A	**4**	**1**	**1**												

Superposition of S.H.M's and Resonance

Q	1	2	3	4	5	6	7	8	9	10	11	12	13
A	**3**	**4**	**3**	**3**	**1**	**3**	**4**	**1**	**2**	**4**	**1**	**3**	**1**